1 The Equations

$$
\begin{aligned}
P(A) + P(\bar{A}) &= 1 \\
P(A) &= P(A \cap B) + P(A \cap \bar{B}) \\
P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\
P(B)P(A|B) = P(A \cap B) &= P(A)P(B|A)
\end{aligned}
$$

1.1 Some special cases

If A and B are independent then:

$$
\begin{aligned}
P(A \cap B) &= P(A)P(B) \\
P(A|B) &= P(A)
\end{aligned}
$$

If $\{B_i\}$ are mutually exclusive events, that cover every possible outcome then:

$$
\begin{aligned}
\sum_{i=1}^{N} P(B_i) &= 1 \\
P(A) &= \sum_{i=1}^{N} P(A \cap B_i)
\end{aligned}
$$

For example:

If d is the result of a rolled die, then $B_i = \{\text{the event } d = i\}$, for $i = 1, 2, \cdots, 6$ are mutually exclusive and cover every possibility.

$B_1 = \{''it's\ raining''\}$ and $B_2 = \{''it's\ lunchtime''\}$ are neither mutually exclusive, nor cover every possibility. (What about sunny breakfast?)

2 Combinatorics

If every possibility is equally probable, then:

$$P(A) = \frac{\#\text{ of ways } A \text{ can happen}}{\#\text{ of ways anything can happen}} \tag{1}$$

3 Mean, Variance, and Expectation

X is a bunch of discrete values x_i with probabilities $P(x_i)$.

$$
\begin{aligned}
\mu = E[X] &= \sum_i x_i P(x_i) \\
\sigma^2 = Var(X) &= \sum_i (x_i - \mu)^2 P(x_i)
\end{aligned}
$$

X is a continuous random variable with PDF P(x).

$$\mu = E[X] = \int xP(x)dx$$
$$\sigma^2 = Var(X) = \int (x-\mu)^2 P(x)dx$$

4 Probability Distribution Functions

	PDF	μ	$\boldsymbol{\sigma^2}$
Bernoulli	$P(i) = \begin{cases} p & i=1 \\ 1-p & i=0 \end{cases}$	p	$p(1-p)$
Binomial	$P(i) = \binom{N}{i} p^i (1-p)^{N-i}$	Np	$Np(1-p)$
Poisson	$P(i) = \frac{e^{-\lambda}}{i!}\lambda^i$	λ	λ
Normal	$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	μ	σ^2
Uniform(disc.)	$P(i) = \{\frac{1}{N}, 1 \leq i \leq N$	$\frac{N+1}{2}$	$\frac{N(N^2-1)}{12}$
Uniform(cont.)	$P(x) = \{\frac{1}{L}, 0 \leq x \leq L$	$\frac{L}{2}$	$\frac{L^3}{12}$

If "Y" and "Z" are independent random variables, with probability distribution functions $P_y(x)$, $P_z(x)$, expectation values μ_y, μ_z, and variances σ_y^2, σ_z^2, and "$Y+Z$" is the random variable created by sampling from Y and Z and adding the results, then the following all hold:

$\mu_{y+z} = \mu_y + \mu_z$
$\sigma_{y+z}^2 = \sigma_y^2 + \sigma_z^2$
$P_{y+z}(i) = \sum_k P_y(k)P_z(i-k) = \sum_k P_y(i-k)P_z(k)$ (for discrete PDFs)
$P_{y+z}(x) = \int P_y(T)P_z(x-T)dt = \int P_y(x-T)P_z(T)dt$ (for continuous PDFs)

Using "$Z_1 + \cdots + Z_N$" to mean sampling from Z, N times, and summing the results yields:

$\mu_{z_1+\cdots+z_N} = N\mu_z$
$\mu_{cz} = c\mu_z$
$\sigma_{cz}^2 = c^2\sigma_z^2$
$\sigma_{z_1+\cdots+z_N}^2 = N\sigma_z^2$
$\sigma_{\frac{1}{N}(z_1+\cdots+z_N)}^2 = \frac{\sigma_z^2}{N}$ "larger averages are more certain"
$\sigma_{\frac{1}{\sqrt{N}}(z_1+\cdots+z_N)}^2 = \sigma_z^2$

4.1 Central Limit Theorem

Noting that $\mu_{z_1+\cdots+z_N} = N\mu_z$ and $\sigma^2_{z_1+\cdots+z_N} = N\sigma^2_z$:

$$Z_1 + \cdots + Z_N \approx \frac{1}{\sqrt{2\pi N\sigma_z^2}} e^{-\frac{(x-N\mu_z)^2}{2N\sigma_z^2}} \quad \text{for large N} \tag{2}$$

Don't stress to much about the π stuff in front. That's part of the "normalization constant" that ensures that the total probability is 1.

For example: Since the binomial distribution is just a sum of samples from the bernoulli distribution, and the poisson distribution is just a kind of binomial distribution:

$$P(x) = \binom{N}{x} p^x (1-p)^{N-x} \approx \frac{1}{\sqrt{2\pi Np(1-p)}} e^{-\frac{(x-Np)^2}{2Np(1-p)}} \tag{3}$$

$$P(x) = \frac{e^{-\lambda}}{x!}\lambda^x \approx \frac{1}{\sqrt{2\pi\lambda}} e^{-\frac{(x-\lambda)^2}{2\lambda}} \tag{4}$$