

# 1 Trigonometry

## 1.1 Pythagorean Identity

$$\sin^2 A + \cos^2 A = 1 \quad (1)$$

## 1.2 Angle Sums

$$\sin(A + B) = \sin A \cos B + \cos A \sin B \quad (2)$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B \quad (3)$$

## 1.3 Definition of Tangent

$$\tan A = \frac{\sin A}{\cos A} \quad (4)$$

## 1.4 Double Angle

These are what you get when you plug in the same thing for both A and B in the angle sum formulas.

$$\sin(2A) = 2 \sin A \cos A \quad (5)$$

$$\cos(2A) = \cos^2 A - \sin^2 A \quad (6)$$

## 1.5 Half Angle

Start with  $\cos(2A) = \cos^2 A - \sin^2 A$ , tie in  $\sin^2 A + \cos^2 A = 1$ , and solve for either sin or cos. For sin A:

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$\cos(2A) = (1 - \sin^2 A) - \sin^2 A$$

$$\cos(2A) = 1 - 2 \sin^2 A$$

$$2 \sin^2 A = 1 - \cos(2A)$$

$$\sin^2 A = \frac{1 - \cos(2A)}{2}$$

$$\sin A = \sqrt{\frac{1 - \cos(2A)}{2}} \quad (7)$$

For cos A:

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$\cos(2A) = \cos^2 A - (1 - \cos^2 A)$$

$$\cos(2A) = 2 \cos^2 A - 1$$

$$2 \cos^2 A = 1 + \cos(2A)$$

$$\cos^2 A = \frac{1 + \cos(2A)}{2}$$

$$\cos A = \sqrt{\frac{1 + \cos(2A)}{2}} \quad (8)$$

## 1.6 Shifting Identities

$$\cos(A \pm 2\pi) = \cos A \quad (9)$$

$$\sin(A \pm 2\pi) = \sin A \quad (10)$$

All that these equations are saying is that if you go all the way around a circle, you'll be in the same place.

$$\cos(A \pm \pi) = -\cos A \quad (11)$$

$$\sin(A \pm \pi) = -\sin A \quad (12)$$

These mean that the  $X$ , and  $Y$  values on opposite sides of a circle are negatives of each other.

$$\cos\left(A - \frac{\pi}{2}\right) = \sin A \quad (13)$$

$$\sin\left(A + \frac{\pi}{2}\right) = \cos A \quad (14)$$

Finally, these you can understand in terms of looking at the graphs of sine and cosine and shifting to the left or right.

## 1.7 Odd and Evenness

$$\cos(-A) = \cos(A) \quad (15)$$

$$\sin(-A) = -\sin(A) \quad (16)$$