Vertical Asymptotes:

When you divide by zero the function blows up, and usually switches positive/negative.

For example:

$F(x) = \sin(x)/(x-1)$ has an asymptote at $x=1$.

$G(x) = x^3/((x+1)(x-2))$ has asymptotes at $x=-1,2$.

$H(x) = (x^2-1)/(x^2+1)$ has no vertical asymptotes.

Zeros:

When the numerator of a function is zero, the entire function is zero. The function can also switch sign at a zero.

For example:

$F(x) = \sin(x)/(x-1)$ has zeros at

$x = \ldots, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots$

$G(x) = x^3/((x+1)(x-2))$ has a zero at $x=0$.

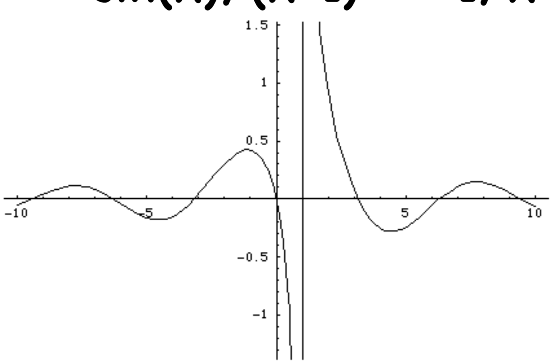
$H(x) = (x^2-1)/(x^2+1)$ has zeros at $x=-1,1$.

Horizontal Asymptotes:

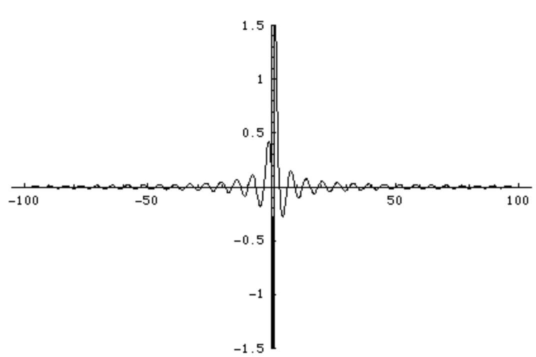
The horizontal asymptote is the value that the function approaches as x goes to ∞ or $-\infty$ (also called long run behavior). You can take the biggest term (with the largest exponent) on the top and bottom, ignore all the others, and compare.
The reasoning is that the largest term "dominates" the smaller terms, so that the contribution they make is tiny. If you look at x^2, x, and 1 when x = one million, then x^2 = one trillion (thousand billions), which is a million times bigger than x. So for large numbers we write "$x^2+x+1 \rightarrow x^2$", with "$\rightarrow$" meaning "approaches" or "behaves like".

F(x) = sin(x)/(x-1) approaches 0 as $x \rightarrow \pm\infty$. sin(x) stays between ±1, but 1/(x-1) drags the function down to zero.

$\sin(x)/(x-1) \rightarrow 1/x \rightarrow 0$



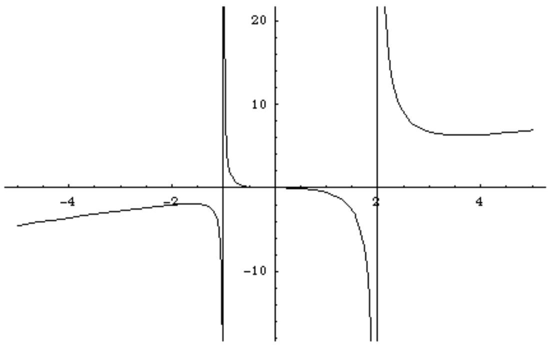
F(x) small scale



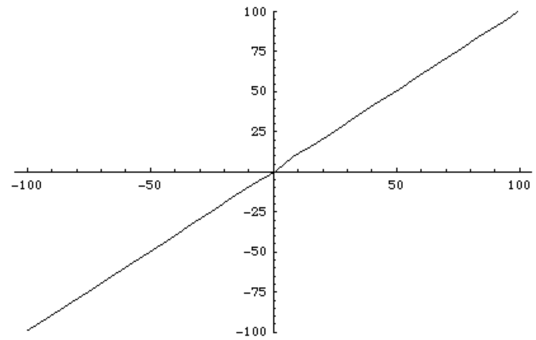
F(x) large scale
(looks like y=0)

G(x) = $x^3/((x+1)(x-2))$ has no horizontal asymptote. Since the top has a higher power of x the function blows up as $x \to \pm\infty$.

$$x^3/((x+1)(x-2)) \to x^3/x^2 = x$$



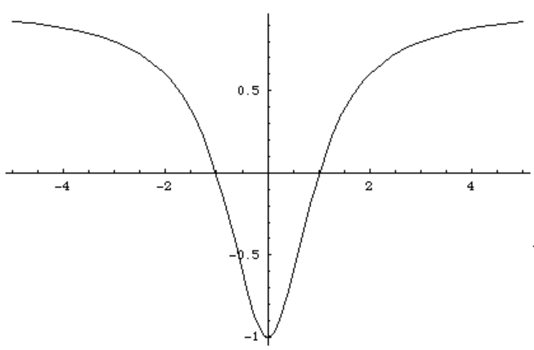
G(x) small scale



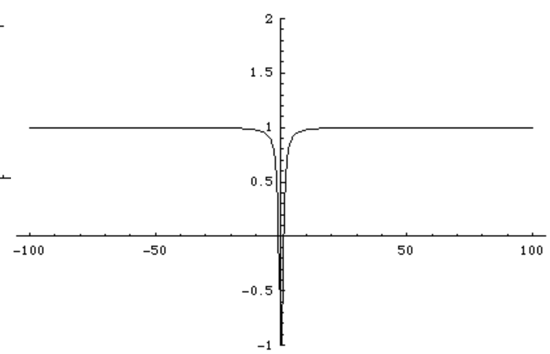
G(x) large scale (looks like y=x)

H(x) = $(x^2-1)/(x^2+1)$ approaches 1 as $x \to \pm\infty$. The highest power on the top and the bottom match.

$$(x^2-1)/(x^2+1) \to x^2/x^2 = 1$$



H(x) small scale



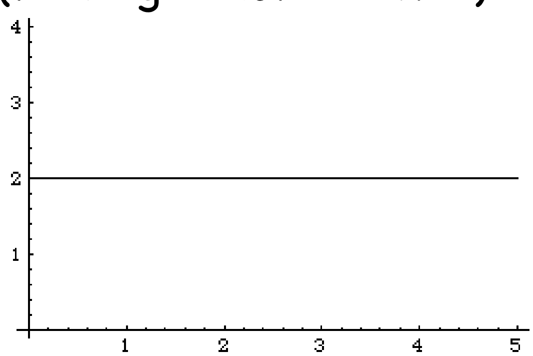
H(x) large scale (looks like y=1)

The Exception:

Zeros on the top and bottom can cancel, but at the same time you can still never divide by zero. So if the <u>degree</u> of the zeros match then they leave a hole in the function. If the bottom zero has a higher degree it's an asymptote. If the top zero has a higher degree it behaves like the function goes to zero, but there's still a hole in the function.

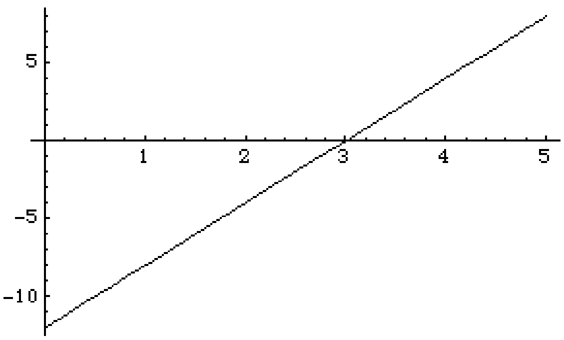
For example:

$J(x) = (2x-6)/(x-3) = 2$, for $x \neq 3$. The top and bottom zero both have degree one and cancel (leaving a hole at $x=3$).



$K(x) = (2x-6)^2/(x-3) = 4(x-3)$, for $x \neq 3$

Tries to be zero, but doesn't quite make it. The top zero has degree 2, the bottom zero has degree 1, so the top "wins" (there's still a hole at $x=3$).



$L(x) = (2x-6)/(x-3)^2 = 2/(x-3)$, for $x \neq 3$

This <u>is</u> a vertical asymptote, since the bottom zero has degree 2, and the top zero has degree 1.