PID Controllers

Control Engineering

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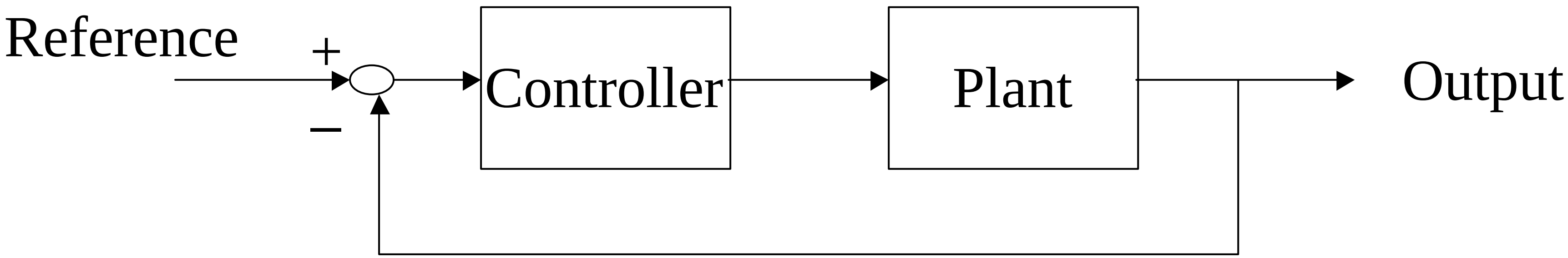
Output Feedback Control Systems



- Feed back only the output signal
  - Easy access
  - Obtainable in practice

PID Controllers

- Proportional controllers
  - pure gain or attenuation
- Integral controllers
  - integrate error
- Derivative controllers
  - differentiate error

Proportional Controller

$$u = K_p e$$

- Controller input is error (reference – output)
- Controller output is control signal
- P controller involves only a proportional gain (or attenuation)

Integral Controller

$$u = K_i \int e dt$$

- Integral of error with a constant gain
- Increase system type by 1
  - Infinity steady-state gain
  - Eliminate steady-state error for a unit step input

Integral Controller

$$\frac{Y(s)}{R(s)} = \frac{G_p(s)}{1+G_p(s)}$$

$$Y(s) = E(s)G_p(s)$$

$$E(s) = \frac{R(s)}{1+G_p(s)}$$

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1+G_p(s)} = \lim_{s\to 0} \frac{1}{1+G_p(s)} = \frac{1}{1+\infty} = 0$$

Derivative Control

$$u = K_d \frac{de}{dt}$$

- Differentiation of error with a constant gain
- Reduce overshoot and oscillation
- Do not affect steady-state response
- Sensitive to noise

Controller Structure

- Single controller
  - P controller, I controller, D controller
- Combination of controllers
  - PI controller, PD controller
  - PID controller

PID Controller

- PI controller $u = K_p e + K_i \int e dt$

- PD controller $u = K_p e + K_d \dfrac{de}{dt}$

- PID controller $u = K_p e + K_i \int e dt + K_d \dfrac{de}{dt}$

PID Controller

- PI controller $U(s) = (K_p + \frac{K_i}{s})E(s)$

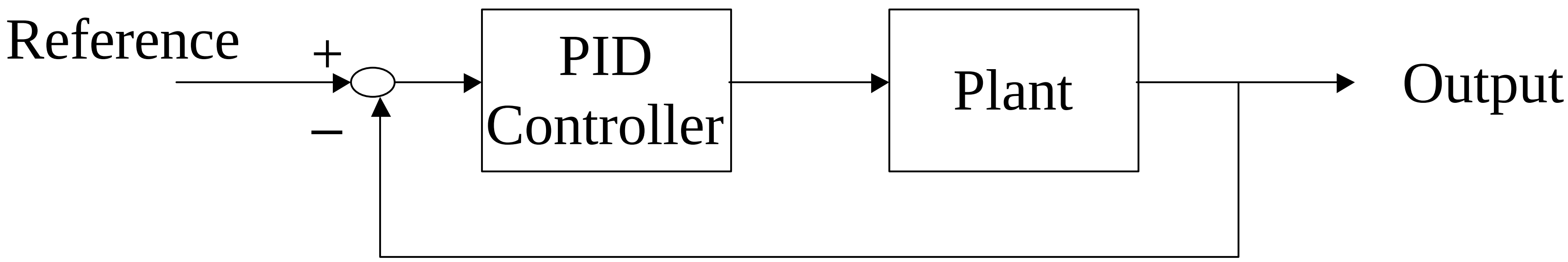
- PD controller $U(s) = (K_p + K_d s)E(s)$

- PID controller $U(s) = (K_p + \frac{K_i}{s} + K_d s)E(s)$

Controller Performance

- P controller
- PI controller
- PD controller
- PID controller

Block Diagram



P Controller

$$G_p(s) = \frac{1}{s^2 + s + 1}$$

$$U(s) = K_p E(s)$$

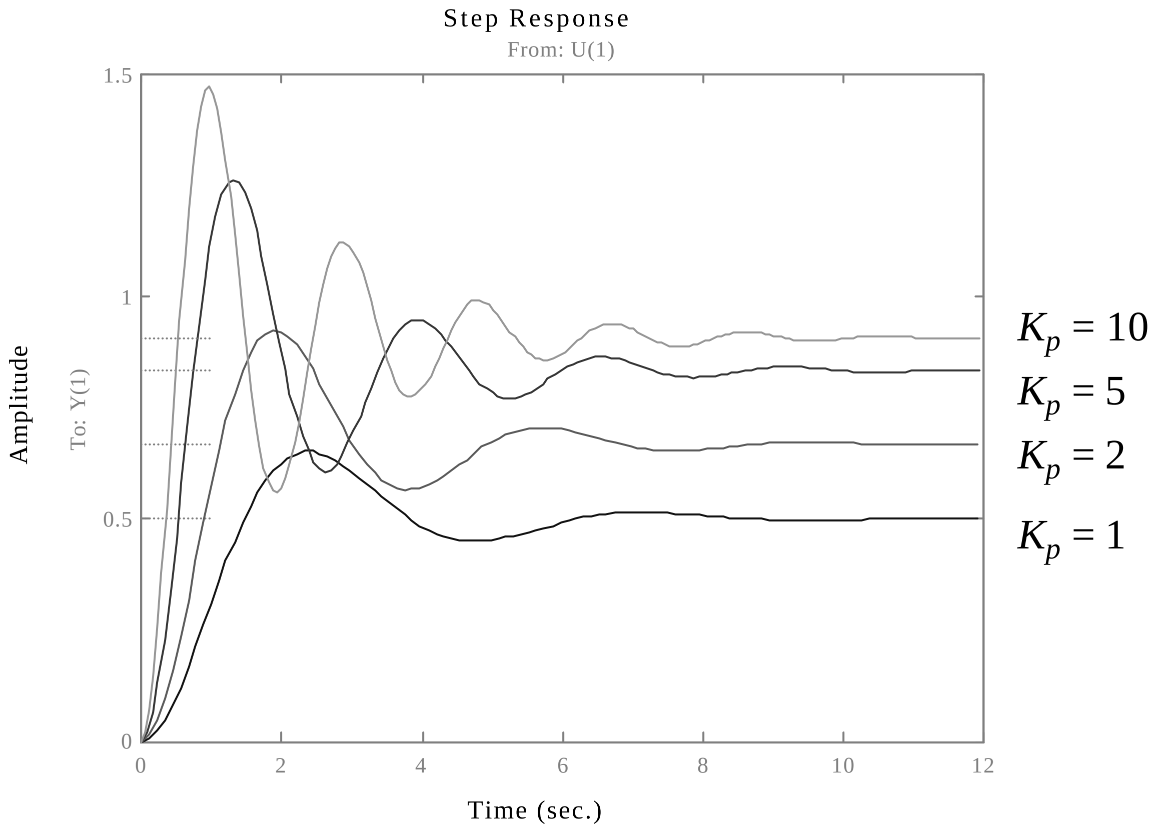
$$OLTF = \frac{K_p}{s^2 + s + 1}$$

$$CLTF = \frac{K_p}{s^2 + s + 1 + K_p}$$

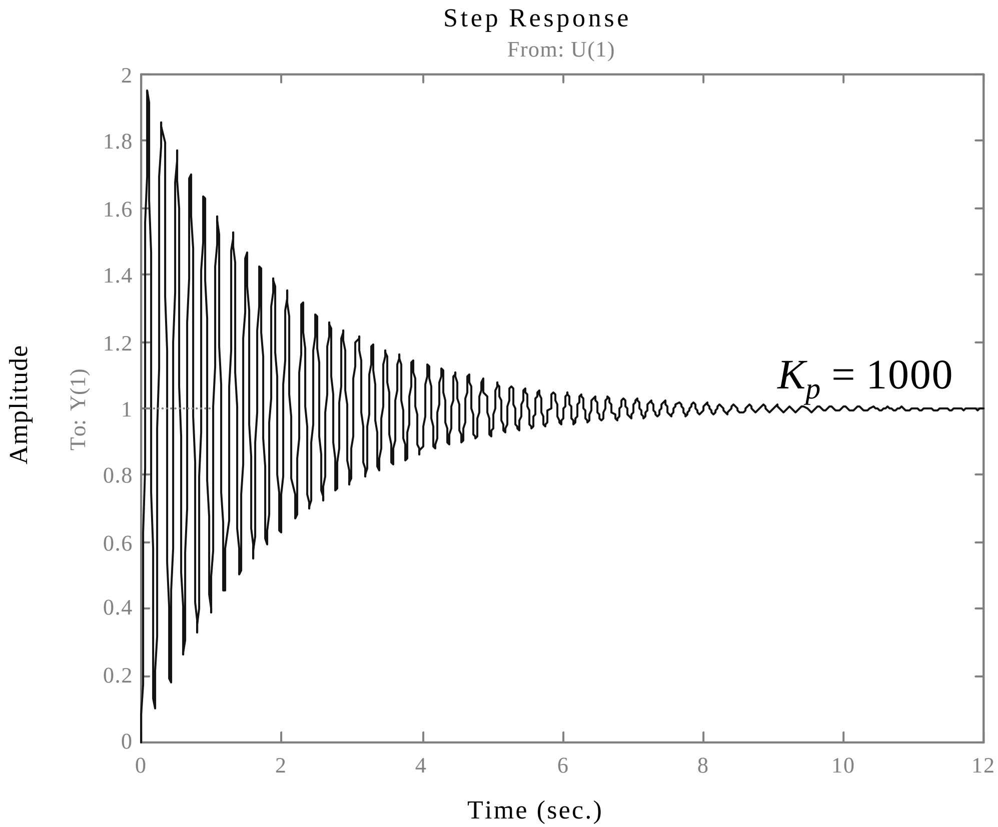
P Controller

- Increase in gain
  - upgrade both steady-state and transient responses
  - reduce steady-state error
  - reduce stability

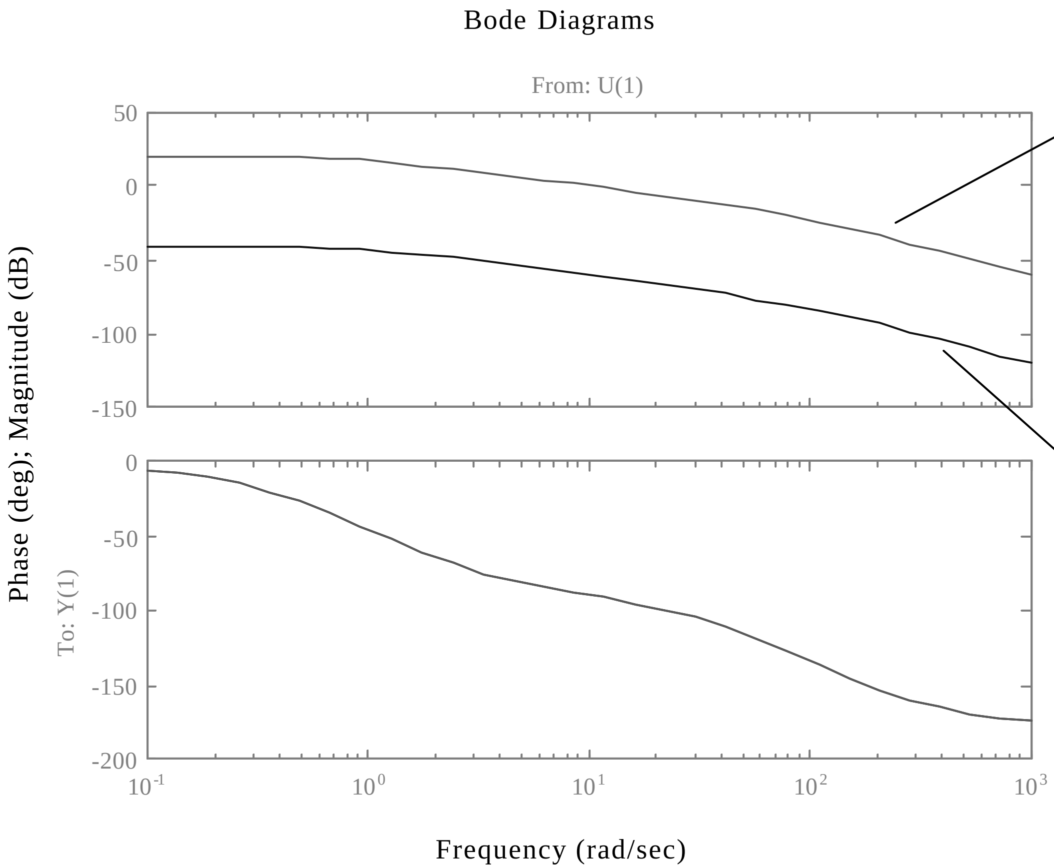
P Controller



P Controller



Proportional Controller



$$L_3(s) = \frac{1000}{(s+1)(s+100)}$$

$$L_2(s) = \frac{1}{(s+1)(s+100)}$$

PI Controller

$$\frac{U(s)}{E(s)} = (K_p + \frac{K_i}{s})$$

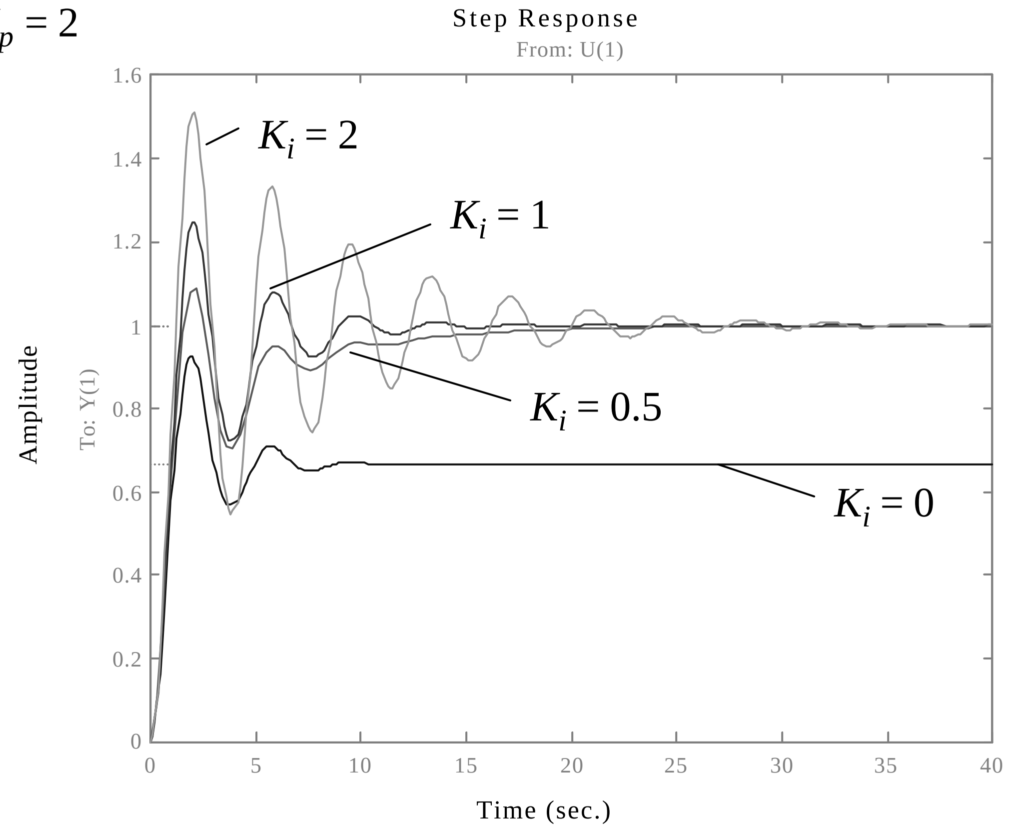
$$\frac{Y(s)}{E(s)} = (K_p + \frac{K_i}{s})G_p(s)$$

$$= \frac{(K_p s + K_i)}{s} G_p(s)$$

$$CLTF = \frac{K_p s + K_i}{s^3 + s^2 + (1 + K_p)s + K_i}$$

PI Controller

$K_p = 2$



PD Controller

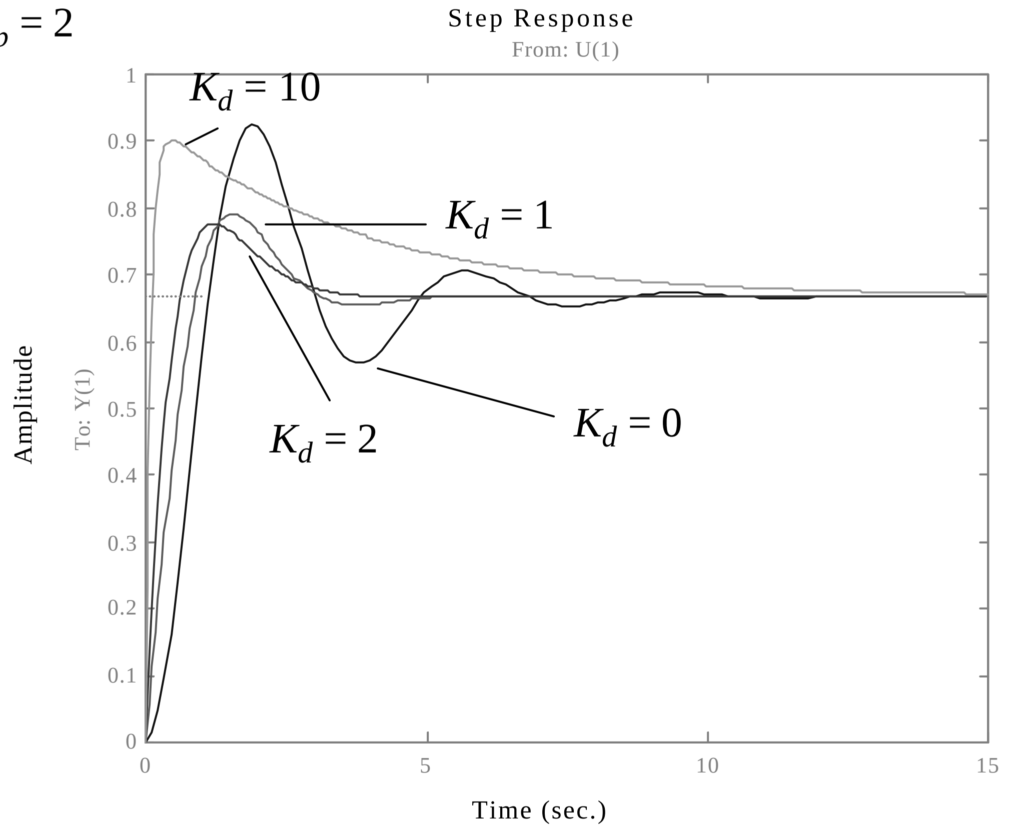
$$U(s) = (K_p + K_d s)E(s)$$

$$OLTF = \frac{K_d s + K_p}{s^2 + s + 1}$$

$$CLTF = \frac{K_d s + K_p}{s^2 + (1 + K_d)s + (1 + K_p)}$$

PD Controller

$K_p = 2$



PID Controller

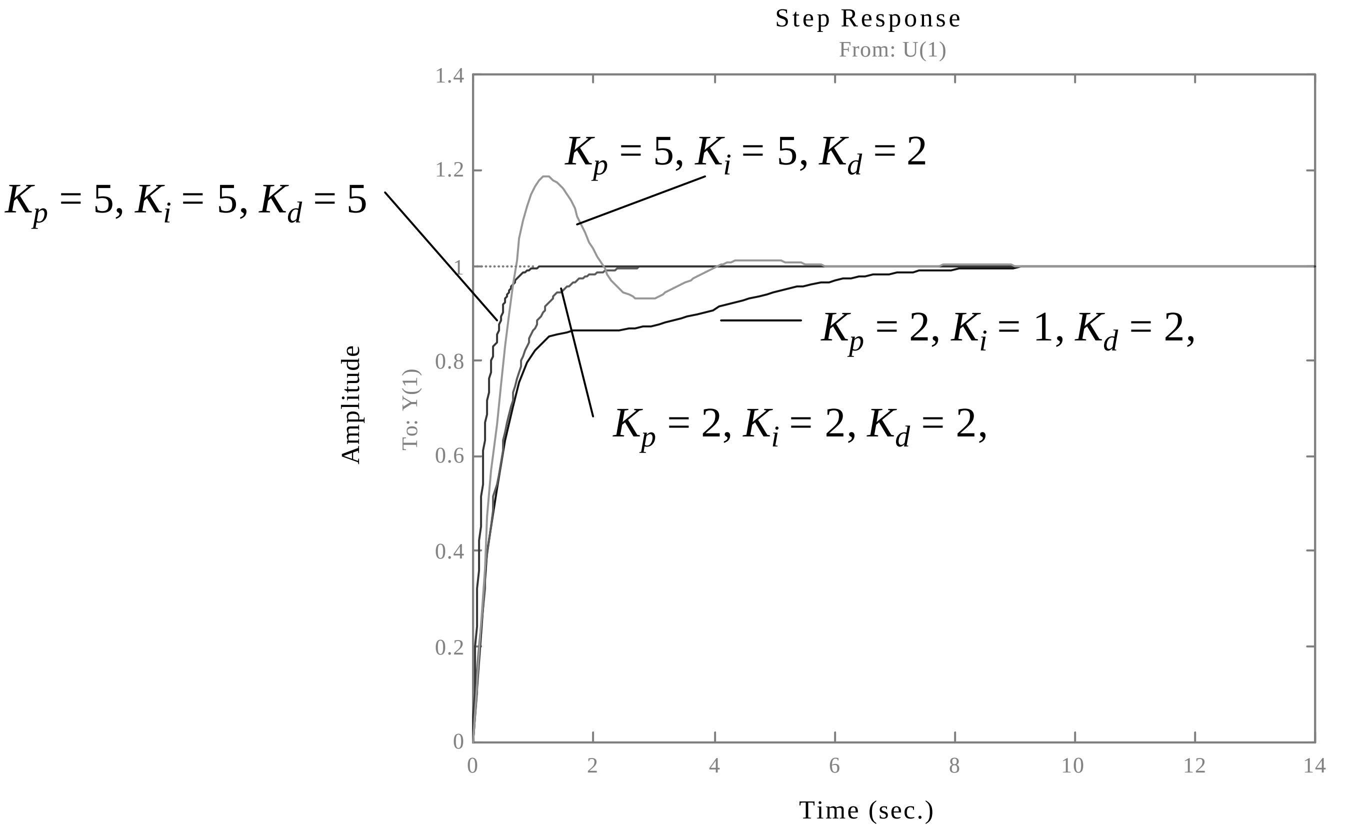
$$U(s) = (K_p + \frac{K_i}{s} + K_d s)E(s)$$

$$= \frac{K_d s^2 + K_p s + K_i}{s} E(s)$$

$$OLTF = \frac{K_d s^2 + K_p s + K_i}{s^3 + s^2 + s}$$

$$CLTF = \frac{K_d s^2 + K_p s + K_i}{s^3 + (1 + K_d)s^2 + (1 + K_p)s + K_i}$$

PID Controller



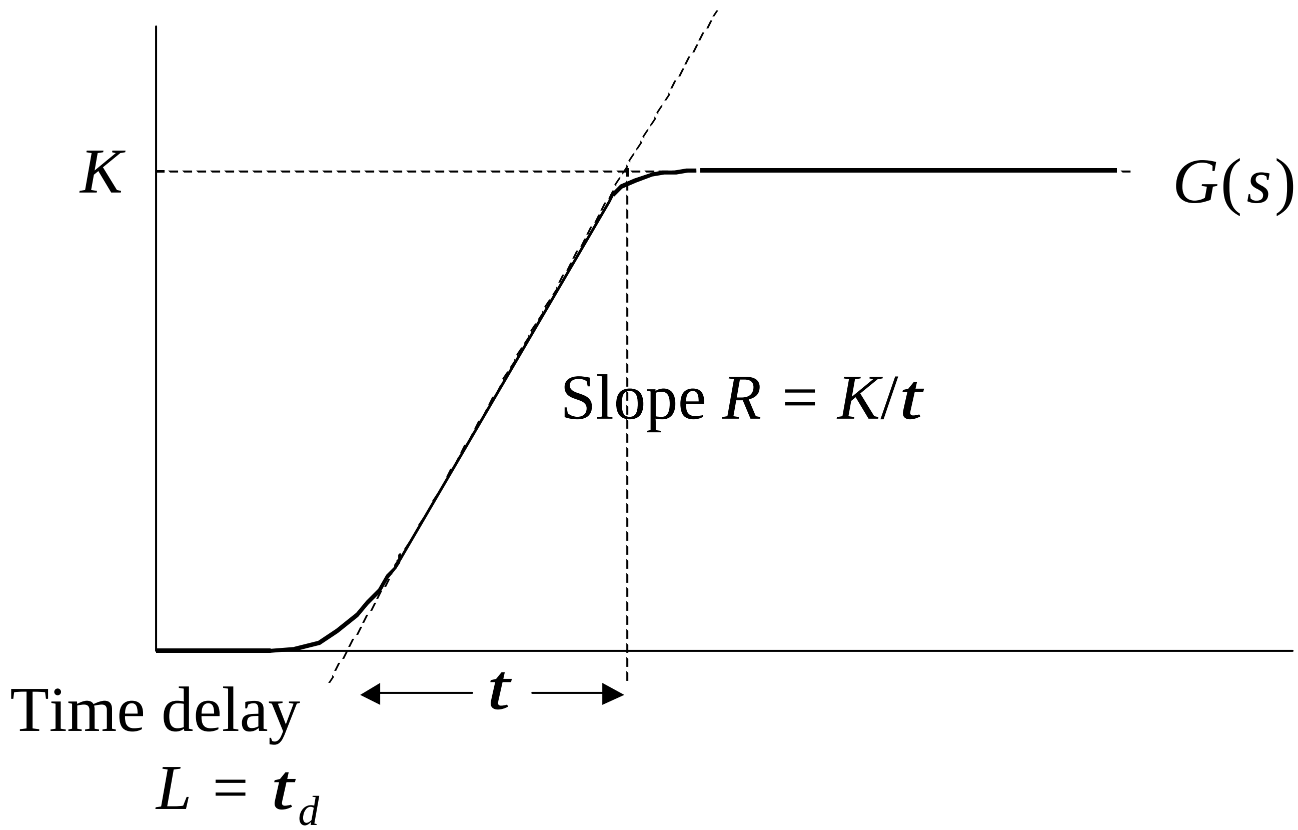
Design of PID Controllers

- Based on the knowledge of P, I and D
  - trial and error
  - manual tuning
  - simulation

Design of PID Controllers

- Ziegler-Nichols method
  - based on a open-loop process
  - based on a critical gain

Ziegler-Nichols Method 1



$$G(s) = \frac{Ke^{-t_d s}}{ts + 1}$$

Ziegler-Nichols Method 1

- P controller
  - $K_p = 1/RL$
- PI controller
  - $K_p = 0.9/RL$, $K_i = 0.27/RL^2$
- PID controller
  - $K_p = 1.2/RL$, $K_i = 0.6/RL^2$, $K_d = 0.6/R$

Ziegler-Nichols Method 2

- Increase a pure gain K_u of a closed-loop system until the system is marginally stable
- Measure the period of oscillation P_u (unit is second)

Ziegler-Nichols Method 2

- P controller
  - $K_p = 0.5K_u$
- PI controller
  - $K_p = 0.45K_u$, $K_i = 0.54K_u/P_u$
- PID controller
  - $K_p = 0.6K_u$, $K_i = 1.2K_u/P_u$, $K_d = 0.075K_uP_u$

Digital P and D Controller

$$U(z) = K_p E(z)$$

$$u(t) = \frac{de(t)}{dt}$$

$$u(k) \approx \frac{e(k) - e(k-1)}{T}$$

Digital I Controller

$$u(t) = u(t_0) + \int_{t_0}^{t} e(\boldsymbol{t})d\boldsymbol{t}$$

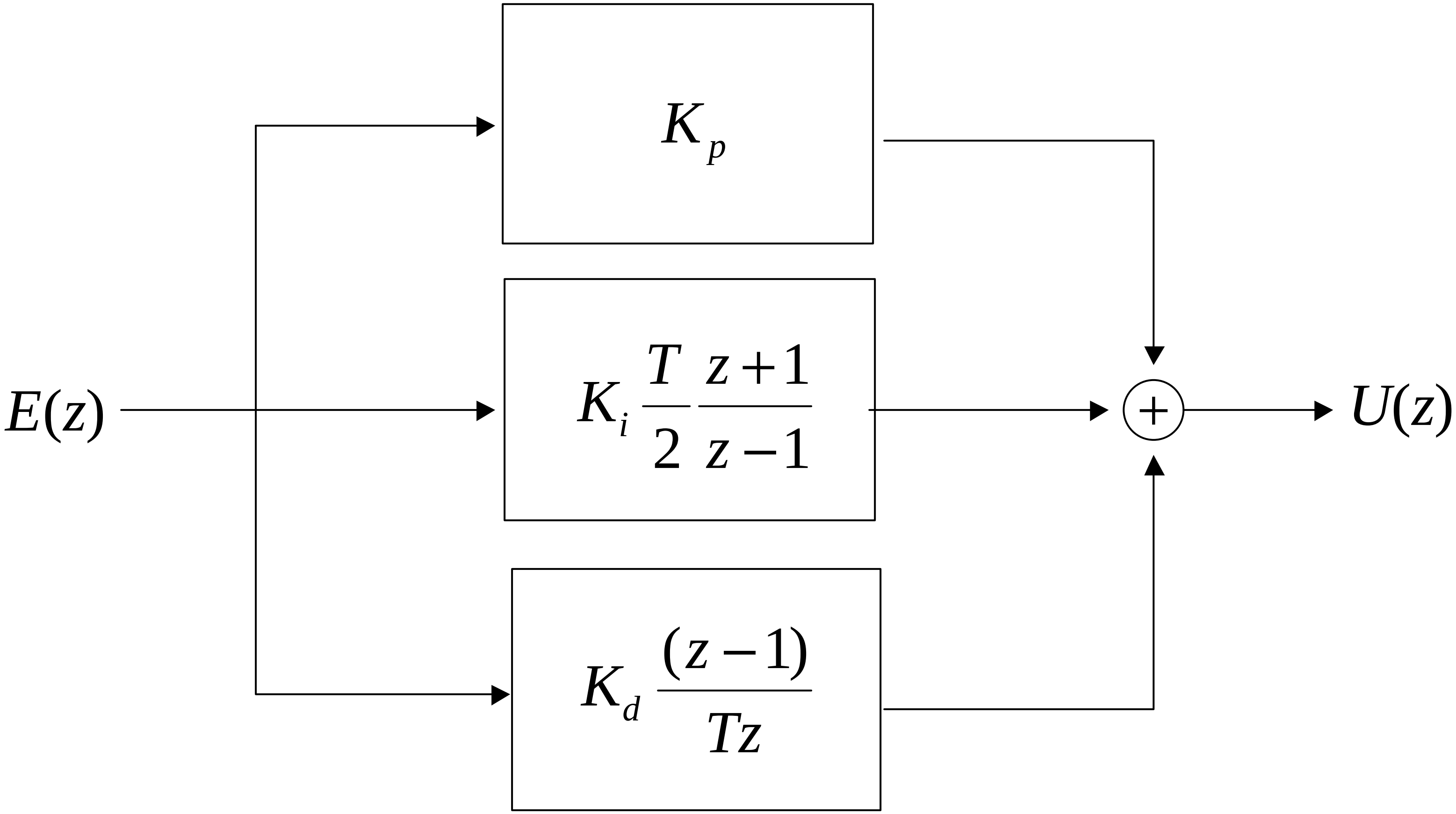
$$u(k+1) = u(k) + \int_{kT}^{(k+1)T} e(\boldsymbol{t})d\boldsymbol{t}$$

$$= u(k) + \frac{T}{2}\left[e(k+1) + e(k)\right]$$

$$zU(z) = U(z) + \frac{T}{2}\left[zE(z) + E(z)\right]$$

$$U(z) = \frac{T}{2}\frac{z+1}{z-1}E(z)$$

Digital PID Controller



Conclusion

- Properties of P, I, D, PI, PD, and PID controllers
- Design of PID controllers
- Digital PID controllers

Reference

- M. Gopal, *Digital Control Engineering*. John Wiley & Sons.
- B. C. Kuo, *Automatic Control System*. Englewood Cliffs, N.J.: Prentice Hall, 1995.
- G. F. Franklin, J. D. Powell, and A. Emami-Naeini, *Feedback Control of Dynamic Systems*. Singapore: Addison-Wesley, 1988.