Transactions Letters

Reduced-State Sequence Estimator with Reverse-Time Structure

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***Abstract*— A modified reduced-state sequencer estimator (RSSE) with a reverse-time structure is proposed for postcursor dominated impulse response channels. This structure can provide significant performance improvements over a conventional RSSE of similar complexity for a well-defined and practically motivated class of channels.**

***Index Terms*— Intersymbol-interference (ISI), MLSE, RSSE, time reversal, time reverse RSSE.**

I. Introduction

When compared with the equalization performance of maximum-likelihood sequence estimation (MLSE) the reduced-state sequence estimation (RSSE) technique exhibits limitations and qualitative differences. The RSSE technique is only well suited to channels where most of the impulse response energy occurs directly after the cursor and any precursor is small [1], [2]. Then, the channels postcursor intersymbol interference (ISI) can be dealt with by a reduced-state trellis search augmented by decision feedback. This means that RSSE can only be used effectively on restricted classes of channels which are largely minimum phase in character (apart from some gross delay). This letter aims at extending the applicability of RSSE to postcursor-dominated channels by using a reverse-time processing structure.

In certain applications, the time-domain impulse response of the channel exhibits a form where significant amounts of the energy occurs at the end rather than the beginning of the response. Wireless local-area network (LAN) channels can take such a profile whenever their direct path is attenuated by obstruction relative to reflections. In [3], Ariyavisitakul develops a decision feedback equalizer (DFE) based on a reverse-time structure for such channels with great success. In this letter, we extend this idea to a RSSE-based on a reverse-time structure.

II. Problem Formulation

Consider a real-valued communication system comprising of the channel and equalizer models in Fig. 1. The channel output at time k is given by

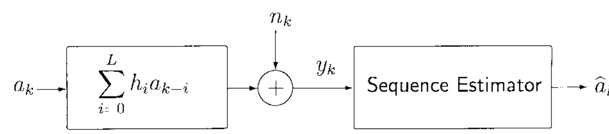

Fig. 1. System and signal model.

$$y_k = \sum_{i=0}^{L} h_i a_{k-i} + n_k \tag{1}$$

where h_i are the finite impulse response channel coefficients, L is the impulse response length excluding the cursor, a_k are the data symbols, and n_k is white zero-mean Gaussian noise with variance

$$N_0 = \sigma_{n_k}^2 = E\{n_k^2\}. \tag{2}$$

The vector of channel impulse response coefficients will be denoted by $\mathbf{h}$. The signal-to-noise ratio (SNR) is defined as

$$\frac{E_s}{N_0} = 10\log_{10}\left(\frac{E\{a_k^2\}\sum_{i=0}^{L} h_i^2}{N_0}\right)\text{dB}. \tag{3}$$

The equalization objective is to estimate the data sequence $\{a_k\}$. Toward this goal, we define the channel state at time k by

$$x_k = (a_{k-1}, a_{k-2}, \cdots, a_{k-L}) \tag{4}$$

which plays a key role in MLSE especially with regard to the Viterbi Algorithm [4].

A. RSSE and Motivation

As a design technique, RSSE as presented in the literature can exploit the set partitioning principles of [1] (whenever the data symbols come from a higher order, usually complex, constellation) as well as exploit state-space channel models [2] (e.g., finite-dimensional recursive models). In what follows, we treat the simplest scenario where the constellation is binary (i.e., binary phase shift keying, BPSK), the channel is of finite duration, as in (1), and the reduced-order state is obtained by truncation of (4) to l components yielding

$$\tilde{x}_k = (a_{k-1}, a_{k-2}, \cdots, a_{k-l}), \quad 0 \le l \le L. \tag{5}$$

The results generalize to cases treated in [1] and [2] without difficulty.

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The full path metric for MLSE, corresponding to a transition from state x_k under input a_k, is given by

$$\lambda(a_k, x_k, y_k) = \left(y_k - \sum_{i=0}^{L} h_i a_{k-i}\right)^2. \qquad (6)$$

In comparison the RSSE path metric, using the truncated state $\tilde{x}_k$, takes the modified form

$$\tilde{\lambda}(a_k, \tilde{x}_k, \tilde{a}(\tilde{x}_k), y_k) = \left(y_k - \sum_{i=0}^{l} h_i a_{k-i} + \sum_{i=l+1}^{L} h_i \tilde{a}_{k-i}(\tilde{x}_k)\right)^2, \quad 0 \le l \le L \qquad (7)$$

where $\tilde{a}(\tilde{x}_k)$ is the survivor sequence which is a function of the reduced state $\tilde{x}_k$. With regard to this survivor, $\tilde{a}_i(\tilde{x}_k)$ is interpreted as the estimate of the data symbol a_i. Clearly, if the dominant channel energy lies in channel coefficients $\{h_{l+1}, h_{l+2}, \cdots, h_L\}$, there will be a significant loss in performance in the RSSE scheme relative to the MLSE scheme. This can also be seen by examining the distance spectra described next.

III. Distance Spectra and Channel Reversal

In motivating a new RSSE processing structure, we reveal a limitation of the conventional RSSE. Define time reversal of a channel as exchanging the first and last, second and second last, and so on, elements of the channel impulse response. The time reversal of channel $\mathbf{h}$ will be denoted by $\bar{\mathbf{h}}$.

A. MLSE Case

The MLSE performance for a time-reversed channel $\bar{\mathbf{h}}$ is identical to that of the channel $\mathbf{h}$ because the distance spectra [5] of $\mathbf{h}$ and $\bar{\mathbf{h}}$ are identical. This follows from the Euclidean distance for an error event ϵ given by

$$d^2_{\bar{\mathbf{h}}}(\bar{\epsilon}) = \sum_{k=k_1(\epsilon)}^{k_2(\epsilon)} \left|\sum_{j=0}^{L} \bar{h}_j \bar{\epsilon}_{k-j}\right|^2 \qquad (8)$$

$$= \sum_{k=k_1(\epsilon)}^{k_2(\epsilon)} \left|\sum_{j=0}^{L} h_{L-j} \epsilon_{k-L+j}\right|^2$$

$$= \sum_{k=k_1(\epsilon)}^{k_2(\epsilon)} \left|\sum_{j=0}^{L} h_j \epsilon_{k-j}\right|^2 = d^2_{\mathbf{h}}(\epsilon) \qquad (9)$$

where $k_1(\epsilon)$ and $k_2(\epsilon)$ are the start and end time indexes for the error event, respectively, and

$$\epsilon_i = \frac{1}{2}(a_i - \hat{a}_i) \in \{-1, 0, +1\}. \qquad (10)$$

Error sequences also satisfy the additional property that they cannot contain more than L consecutive zeros [5]. Let $\mathcal{E}$ denote the set of all such valid error sequences. Then, the distance spectra for $\mathbf{h}$ and $\bar{\mathbf{h}}$ are identical because $\epsilon \in \mathcal{E} \Rightarrow \bar{\epsilon} \in \mathcal{E}$.

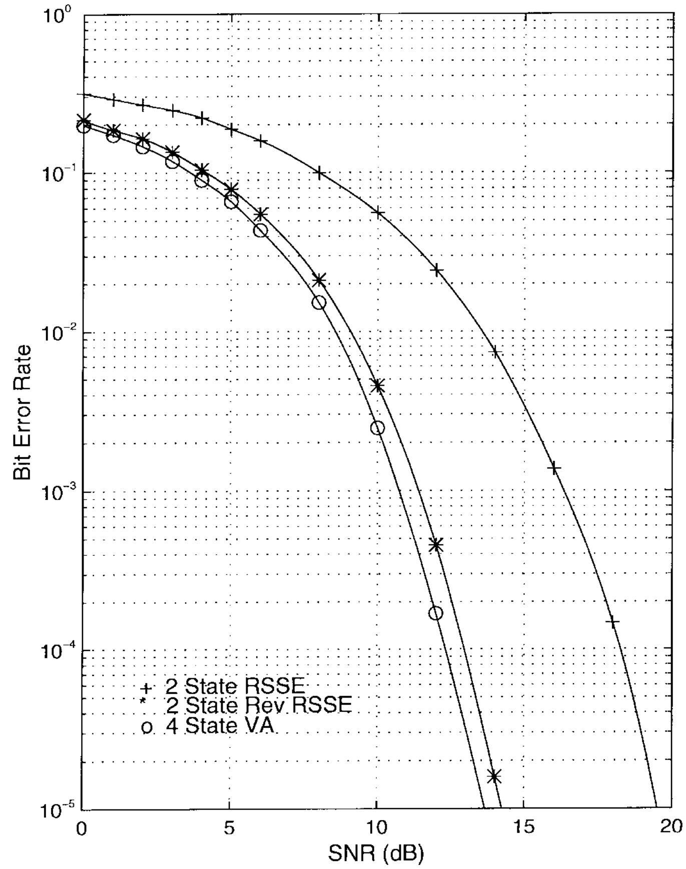

Fig. 2. BER comparison between, MLSE, RSSE, and time-reversed RSSE for channel $\mathbf{h} = (0.3239,\ 0.5831,\ 0.7450)$.

B. RSSE Case

Unlike the MLSE property of (9), the distance spectrum for RSSE breaks the symmetry between an arbitrary channel $\mathbf{h}$ and its time reversal $\bar{\mathbf{h}}$. Techniques to determine the distance spectrum for RSSE or other performance measures can be found in [6] and references therein. This lack of invariance to channel reversal can be crippling in terms of the performance.

For example, consider the simple closed eye unity energy normalized channel:

$$\mathbf{h} = (0.3239,\ 0.5831,\ 0.7450) \qquad (11)$$

where $L = 2$ and with $l = 1$, i.e., there are $2^L = 4$ MLSE states and $2^l = 2$ reduced states. Then, by straightforward but tedious calculation, we have the following minimum Euclidean distance results:

$$\text{Four State MLSE: } \min_{\epsilon\in\mathcal{E}} d^2_{\mathbf{h}}(\epsilon) = \min_{\epsilon\in\mathcal{E}} d^2_{\bar{\mathbf{h}}}(\epsilon) = 0.8679 \qquad (12)$$

$$\text{Two State RSSE: } \min_{\epsilon\in\mathcal{E}} d^2_{\mathbf{h}}(\epsilon) = 0.4453 \qquad (13)$$

$$\text{Two State RSSE: } \min_{\epsilon\in\mathcal{E}} d^2_{\bar{\mathbf{h}}}(\epsilon) = 0.8052. \qquad (14)$$

We also provide the full distance spectra in Fig. 3, where the vertical scale is the error multiplicity which takes into account the probability of the error sequences. The bit-error performance curves are shown in Fig. 2 and we can see that there is a difference of 5.2 dB at a bit-error rate (BER) of 10^{-5}.

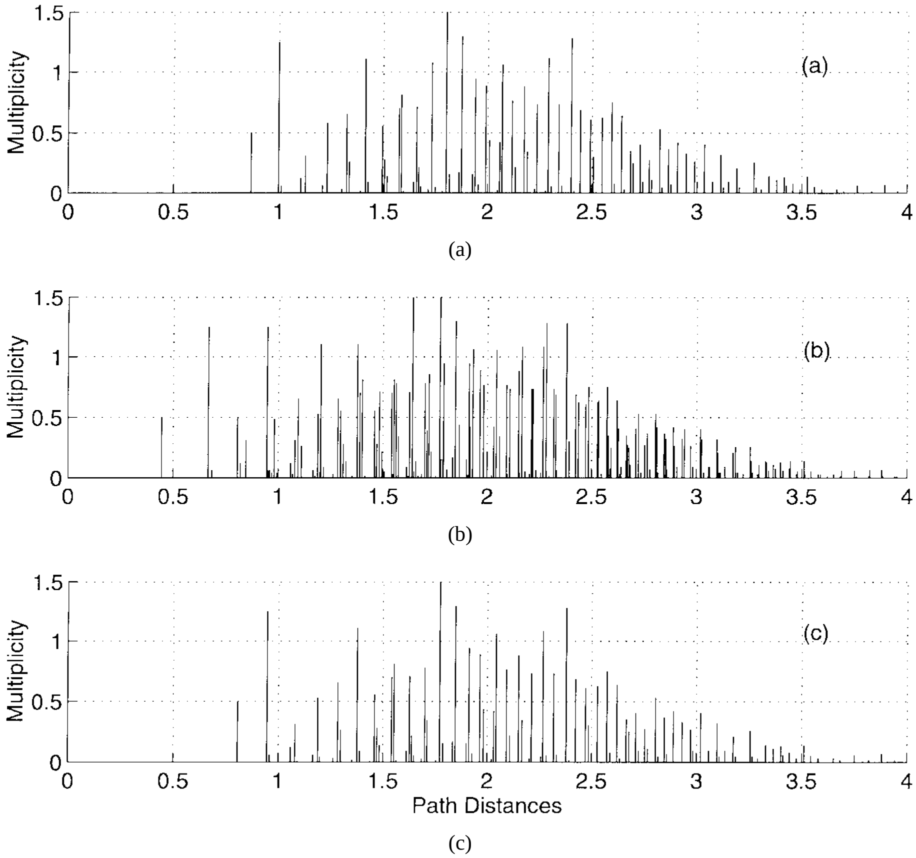

Fig. 3. Euclidean distance spectra for: (a) four-state MLSE, (b) conventional two-state RSSE, and (c) two-state time-reversed RSSE, for channel $\mathbf{h} = (0.3239,\ 0.5831,\ 0.7450)$.

The usual explanation for the significant loss in performance for the RSSE is that the reduction in states promotes early merging and thus there is a greater chance that the MLSE path is prematurely discarded.

IV. Reverse-Time Structure

Following and extending Ariyavisitakul [3], we propose a reverse-time structure for recovering the performance of the RSSE algorithm on channels $\mathbf{h}$ for which the following condition holds:

$$\min_{\epsilon\in\mathcal{E}} d^2_{\mathbf{h}}(\epsilon) \text{ (using conventional RSSE)} < \min_{\epsilon\in\mathcal{E}} d^2_{\bar{\mathbf{h}}}(\epsilon) \text{ (using conventional RSSE)} \qquad (15)$$

i.e., the postcursor has the majority of energy in the channel (we refer to this as postcursor dominated). Whenever (15) does not hold, the conventional RSSE is superior.

A. Performance

The performance of such a modified RSSE structure for channel $\mathbf{h}$, which we call the reverse RSSE, is identical to the conventional RSSE acting on the hypothetical channel $\bar{\mathbf{h}}$, i.e.,

$$\min_{\epsilon\in\mathcal{E}} d^2_{\mathbf{h}}(\epsilon) \text{ (Reverse RSSE)} = \min_{\epsilon\in\mathcal{E}} d^2_{\bar{\mathbf{h}}}(\epsilon) \text{ (Conventional RSSE).} \qquad (16)$$

In addition, not only is the minimum distance the same but also the complete distance spectra. Therefore, the performances are identical. In the limiting case where $l = 0$ (corresponding to a single reduced state) the proposed structure reduces to that presented by Ariyavisitakul [3].

B. Implementation

The reverse-time structure reverses the order of a block of channel output measurements of sufficient length and uses this as data in a conventional RSSE algorithm, but using the reverse of the channel model. The trellis dimensionality and structure are determined in the same manner as for conventional RSSE. Blocks of data estimates at the output need to be reversed as a final step of the processing. Such a structure is well suited to packet-based protocols but can also be extended to operate on a continuous data stream. Grouping of data into blocks is required when applying to a continuous data stream, these blocks require overlapping which consequently adds slightly to the computational overheads.

V. Conclusion

A potential deficiency of RSSE whereby its performance is not invariant to channel reversal can be turned to an advantage for a certain class of channels. Using a reverse-time processing structure for postcursor-dominated channels, significant performance improvements are possible over the conventional RSSE with the same number of states. In a simple example, the performance improvement was 5.2 dB at BER of 10^{-5}.

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