Characterization of 3D Spatial Wireless Channels

Thushara D. Abhayapala, Tony S. Pollock, and Rodney A. Kennedy,
Department of Telecommunications Engineering, RSISE
Australian National University, Canberra ACT 0200 Australia.
{thush,tony,rod}@syseng.anu.edu.au

***Abstract*— In this paper a novel three dimensional spatial channels model is developed to provide insight into spatial aspects of multiple antenna communication systems. The spherical harmonic representation of wavefields is used to decompose the spatial channel matrix into a product of known and random matrices where the known portion shows the effects of the physical configuration of antenna elements. The model supports any arbitrary antenna array configurations as well as any distribution of scatterers. Possible applications of the model and its usefulness are outlined.**

I. Introduction

Multi antenna element wireless communications systems have shown significantly large capacity gains compared to traditional single input single output systems [1], [2]. However, recent results [3], [4] indicate that such large gains are due to overly simplified assumptions of the spatial wireless channels. Hence, proper understanding of the limitations of MIMO systems as well as the maximal exploitation of the spatial dimension need better modelling of the channel. In this paper, we introduce a 3D spatial model based on spatial basis function decomposition of multipath wave fields.

Recent work related to better understanding of MIMO systems are reported in [3–8]. The spatial channel models proposed in [7] and [8] are more general and realistic than the usual independent models, however they are based on uniform linear array antennas and discrete set of scatterers around the transmitter and the receiver arrays limiting their generality. The proposed spatial channel model in this paper is valid for arbitrary 3D antenna arrays and based on decomposition of spatial wavefield using spatial basis functions.

The contribution of this work is the decomposition of a general spatial channel model which includes physical parameters of antenna configurations and a tractable parameterization of complex scattering environment, to overcome the above mentioned deficiencies in the traditional models. We separate the physical spatial channel into three regions of interest: (i) scatterer free region around the transmitter antennas, (ii) scatterer free region around the receiver antennas, and (iii) complex scattering media which is the complement of the union of regions in (i) and (ii). We use the underlying physics of the free-space propagation to model the channel in regions (i) and (ii), and the complex scattering media (iii) is represented by a parametric model. With this separation of the physical channel, we are able to decompose the channel matrix of a MIMO system into a product of three matrices, where two of them are fixed and known for a given antenna orientations and the other representing the parameters of the scattering media (iii).

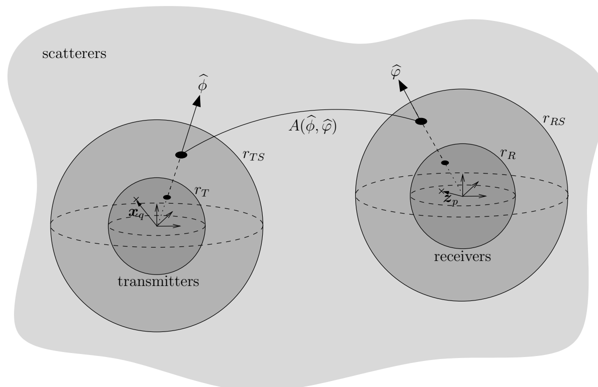

Fig. 1. A general scattering model for a flat fading MIMO system. r_{T} and r_{R} are the radii of spheres which enclose the transmitter and the receiver arrays. $A(\widehat{\boldsymbol{\phi}}, \widehat{\boldsymbol{\varphi}})$ represents the gain of the complex scattering environment for signals leaving the transmitter scattering free region from direction $\widehat{\boldsymbol{\phi}}$ and arriving at the receiver scattering free region from direction $\widehat{\boldsymbol{\varphi}}$.

II. Spatial channel model

Consider a MIMO system with Q transmit antennas located at positions $\boldsymbol{x}_q$, $q = 1, 2, \ldots, Q$ from a transmitter origin and P receiver antennas located at positions $\boldsymbol{z}_p$, $p = 1, 2, \ldots, P$ from a receiver origin, within balls of radius r_{T} and r_{R} respectively, as shown in Fig.1. We assume that scatters are distributed outside the balls of radii r_{TS} ($> r_{\mathrm{T}}$) and r_{RS} ($> r_{\mathrm{R}}$). Thus, the wireless channel has three spatial regions, namely, scattering free balls encompassing transmit antennas and receiver antennas, and the rest of the space assumed to be a complex scattering media. We assume that the surface of scattering free ball is in the farfield of either receiver or transmitter origin.

In this paper, we only consider flat fading channel environment where propagation delay is always less than the symbol period. Our attention is aimed to understand fading due to spatial effects rather than temporal effects.

Let $\boldsymbol{u} = [u_1, u_2, \ldots, u_Q]'$ be the vector of baseband transmitted signals from the Q transmitters during a signaling interval, $A(\widehat{\boldsymbol{\phi}}, \widehat{\boldsymbol{\varphi}})$ is the complex gain of a signal leaving from the transmitter-scattering-free ball at an angle $\widehat{\boldsymbol{\phi}}$ (unit vector) and entering the receiver-scattering-free ball at an angle $\widehat{\boldsymbol{\varphi}}$ (unit vector), $\boldsymbol{v} = [v_1, v_2, \ldots, v_P]'$, be the vector of baseband received signal during a signaling interval, and

$\boldsymbol{n} = [n_1, n_2, \ldots, n_P]'$, be the vector of noise at the receiver antennas where $[\cdot]'$ denotes the vector transpose. Then we can write

$$\boldsymbol{v} = \boldsymbol{H}\boldsymbol{u} + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{H}$ is a $P \times Q$ channel matrix with the (p, q) element given by

$$\{\boldsymbol{H}\}_{pq} = \int_\Omega \int_\Omega A(\widehat{\boldsymbol{\phi}}, \widehat{\boldsymbol{\varphi}})\, e^{ik\boldsymbol{x}_q\cdot\widehat{\boldsymbol{\phi}}}\, e^{-ik\boldsymbol{z}_p\cdot\widehat{\boldsymbol{\varphi}}}\, d\Omega(\widehat{\boldsymbol{\phi}})\, d\Omega(\widehat{\boldsymbol{\varphi}}). \tag{2}$$

where $k \triangleq 2\pi/\lambda$ is the wavenumber, λ the wavelength, $d\Omega(\widehat{\boldsymbol{\phi}})$ and $d\Omega(\widehat{\boldsymbol{\varphi}})$ are surface elements of the unit sphere Ω.

III. Modal Decomposition

In this section, we use basis functions defined on the surface of a sphere to reveal the underlying structure of the channel matrix $\boldsymbol{H}$. This decomposition can be generalized but the advantage of the use of spheres is the existence of explicit bounds.

A. Spherical Harmonic Expansion

In a 3D propagation environment, a plane wave can be expanded [9] as

$$e^{ik\boldsymbol{x}\cdot\widehat{\boldsymbol{y}}} = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} i^n 4\pi j_n(k\|\boldsymbol{x}\|) Y_{nm}(\widehat{\boldsymbol{x}}) \overline{Y_{nm}}(\widehat{\boldsymbol{y}}) \tag{3}$$

where $n \in \mathbb{Z}$, $m \in \mathbb{Z}$, $|m| \le n$, $j_n(\cdot)$ are the spherical Bessel functions of order n, and $Y_{nm}(\cdot)$ are the spherical harmonics

$$Y_{nm}(\triangleq)\sqrt{\frac{2n+1}{4\pi}\frac{(n-|m|)!}{(n+|m|)!}}\, P_n^{|m|}(\cos\theta)\, e^{im\phi}, \tag{4}$$

which form a complete orthonormal function basis set on the unit sphere with respect to the inner product

$$\langle f, g\rangle_{\mathbb{S}^2} \triangleq \int_{\mathbb{S}^2} f(\widehat{\boldsymbol{x}})\overline{g(\widehat{\boldsymbol{x}})}\, ds(\widehat{\boldsymbol{x}}), \tag{5}$$

$$\equiv \int_0^\pi \int_0^{2\pi} f(\theta,\phi)\,\overline{g(\theta,\phi)}\, \sin\theta\, d\phi\, d\theta, \tag{6}$$

$P_n^m(\cdot)$ are the associated Legendre functions, and $\mathbb{S}^2$ is the unit sphere defined by

$$\mathbb{S}^2 \triangleq \{\boldsymbol{x} \in \mathbb{R}^3 \colon \|\boldsymbol{x}\| = 1\}. \tag{7}$$

Using the spatial high pass character of spherical Bessel functions, above series, (3), can be truncated to a well-defined a finite number of terms summarized in the following theorem.

Theorem 1: For a finite $x = \|\boldsymbol{x}\|$, the series (3) can be truncated to $|n| \le N$ with an absolute error

$$\epsilon_N(x) = \Big|\sum_{n>N} \sum_{m=-n}^{n} i^n 4\pi j_n(kx) Y_{nm}(\widehat{\boldsymbol{x}})\overline{Y_{nm}}(\widehat{\boldsymbol{y}})\Big| \tag{8}$$

which, for $N = \lceil ekx/2 \rceil + \Delta$, is bounded by

$$\epsilon_N(x) \le \eta \exp(-\Delta) \tag{9}$$

for an integer $\Delta \ge 0$, where $\eta \approx 0.67848124300079\ldots$.

Proof: We can bound the truncation error (8)

$$\epsilon_N(x) \le 4\pi \sum_{n>N} \big|j_n(kx)\big| \cdot \Big|\sum_{m=-n}^{n} Y_{nm}(\widehat{\boldsymbol{x}})\overline{Y_{nm}}(\widehat{\boldsymbol{y}})\Big| \tag{10a}$$

$$\le \sum_{n>N} (2n+1)\big|j_n(kx)\big| \tag{10b}$$

$$\le \sqrt{\pi} \sum_{n>N} \frac{(\pi x/\lambda)^n}{\Gamma(n+1/2)} \tag{10c}$$

$$\le \frac{1}{\sqrt{2e}} \sum_{n>N} \Big(\frac{\pi e x/\lambda}{n-1/2}\Big)^n \tag{10d}$$

where we have used: i) identity

$$\sum_{m=-n}^{n} Y_{nm}(\widehat{\boldsymbol{x}})\overline{Y_{nm}}(\widehat{\boldsymbol{y}}) = \frac{2n+1}{4\pi} P_n(\widehat{\boldsymbol{x}}\cdot\widehat{\boldsymbol{y}}) \tag{11}$$

where $P_n(\cdot)$ is the Legendre polynomial of order n, and

$$|P_n(\widehat{\boldsymbol{x}}\cdot\widehat{\boldsymbol{y}})| \le 1, \tag{12}$$

in going from (10a) to (10b) [9]; ii) the spherical Bessel function bound

$$j_n(z) \le \frac{\sqrt{\pi}}{2}\frac{1}{\Gamma(n+3/2)}\Big(\frac{z}{2}\Big)^n \equiv \frac{z^n}{(2n+1)!!}, \tag{13}$$

in going from (10b) to (10c); and iii) the Stirling bound on the Gamma function in the form

$$\Gamma(n+1/2) \ge \sqrt{2\pi e}\,\Big(\frac{n-1/2}{e}\Big)^n, \tag{14}$$

in going from (10c) to (10d). The form of (10d) is used later to find an appropriate choice of N. However, first we use (10c) to develop an expression for a suitable bound.

To investigate how $\epsilon_N(x)$ behaves with N it suffices to characterize the remainder

$$R_N(z) \triangleq \sum_{n>N} \frac{z^n}{\Gamma(n+1/2)}, \quad z > 0 \tag{15}$$

noting, $\epsilon_N(x) \le \sqrt{\pi} R_N(\pi e x/\lambda)$ is a restatement of the bound (10c) when we identify $z \triangleq \pi e x/\lambda$.

First we observe that

$$R_N(z) = \frac{z^{N+1}}{\Gamma(N+3/2)}\Big(1 + \frac{z}{N+5/2} + \frac{z^2}{(N+5/2)(N+7/2)} + \cdots\Big) \tag{16a}$$

$$< \frac{z^{N+1}}{\Gamma(N+3/2)}\Big(1 + \frac{z}{N+5/2} + \frac{z^2}{(N+5/2)^2} + \cdots\Big) \tag{16b}$$

$$= \frac{z^{N+1}}{\Gamma(N+3/2)}\Big(\frac{N+5/2}{N+5/2-z}\Big) \tag{16c}$$

$$\le \frac{1}{\sqrt{2\pi e}}\Big(\frac{ze}{N+1/2}\Big)^{N+1}\Big(\frac{N+5/2}{N+5/2-z}\Big) \tag{16d}$$

using the Striling bound (14) and we have assumed $z < N + 5/2$ in the last two steps, so that there is a finite upper bound provided N is large enough. Further by considering the ratio

$$\frac{R_{N+1}(z)}{R_N(z)} = \frac{z^{N+2}}{\Gamma(N+5/2)} \cdot \frac{\Gamma(N+3/2)}{z^{N+1}} \cdot \left(\frac{1 + \frac{z}{N+7/2} + \frac{z^2}{(N+7/2)(N+9/2)} + \cdots}{1 + \frac{z}{N+5/2} + \frac{z^2}{(N+5/2)(N+7/2)} + \cdots} \right) \quad (17a)$$

$$\leq \frac{z}{(N+3/2)} \quad (17b)$$

and more generally

$$\frac{R_{N+\Delta}(z)}{R_N(z)} \leq \frac{z}{(N+3/2)} \cdots \frac{z}{(N+\Delta+1/2)} \quad (18a)$$

$$\leq \left(\frac{z}{N+3/2}\right)^{\Delta} \quad (18b)$$

which establishes that the error has a bound that decreases exponentially provided N is sufficiently large relative to z.

To have an effective bound we need to choose N such that the bound is small in absolute terms as well as having an exponential decay of the error as Δ increases. Guided by (10d) we consider a critical value of N defined as through the integer function

$$N^{\text{crit}}(z) \triangleq \lceil ez \rceil \geq ez, \quad z > 0. \quad (19)$$

Then using simple inequality arguments we have for $N \geq N^{\text{crit}}(z)$

$$\frac{z}{N+3/2} < \frac{z}{N} \leq \frac{1}{e}, \quad \text{for } N > \lceil ez \rceil \quad (20a)$$

$$\frac{N+5/2}{N+5/2-z} \leq \frac{e}{e-1}, \quad \text{for } N > \lceil ez \rceil \quad (20b)$$

$$\frac{ze}{N+1/2} < 1, \quad \text{for } N > \lceil ez \rceil \quad (20c)$$

which implies, from (21) and (20a), an explicit rate of exponential decay

$$\frac{R_{N+\Delta}(z)}{R_N(z)} \leq \exp(-\Delta), \quad \text{for } N > \lceil ez \rceil \quad (21)$$

and from (16d), (20b) and (20c)

$$R_N(z) < \frac{1}{\sqrt{2\pi e}} \cdot \frac{e}{e-1}, \quad \text{for } N > \lceil ez \rceil \quad (22)$$

which defines an explicit uniform bound. Hence combining (21) and (22)

$$\epsilon_N(x) < \frac{\sqrt{e/2}}{e-1} \cdot \exp(-\Delta), \quad \text{for } N > \lceil \pi e x/2 \rceil + \Delta \quad (23)$$

which is (9). ■

B. Channel Decomposition

Using (3), Theorem 1, and (2) we decompose the channel matrix as

$$\boldsymbol{H} = \boldsymbol{J}_{\text{R}} \boldsymbol{H}_{\text{s}} \boldsymbol{J}_{\text{T}}^{\dagger}, \quad (24)$$

where $\cdot^{\dagger}$ represent the conjugate transpose operator, $\boldsymbol{J}_{\text{R}}$ is a $P \times (N_{\text{R}}+1)^2$ matrix whose pth row is given by $[\mathcal{J}_{00}(\boldsymbol{x}_p)\mathcal{J}_{1(-1)}(\boldsymbol{x}_p)\ldots\mathcal{J}_{N_{\text{R}}(-N_{\text{R}})}(\boldsymbol{x}_p)\ldots\mathcal{J}_{N_{\text{R}}N_{\text{R}}}(\boldsymbol{x}_p)]$, where $N_{\text{R}} = \lceil ker_{\text{R}}/2 \rceil + \Delta$,

$$\mathcal{J}_{nm}(\boldsymbol{x}) \triangleq 4\pi i^n\, j_n(k\|\boldsymbol{x}\|)\overline{Y_{nm}}(\widehat{\boldsymbol{x}}), \quad (25)$$

$\boldsymbol{J}_{\text{T}}$ is a $Q \times (N_{\text{T}}+1)^2$ matrix whose qth row is given by $[\mathcal{J}_{00}(\boldsymbol{x}_q)\mathcal{J}_{1(-1)}(\boldsymbol{x}_q)\ldots\mathcal{J}_{N_{\text{T}}(-N_{\text{T}})}(\boldsymbol{x}_q)\ldots\mathcal{J}_{N_{\text{T}}N_{\text{T}}}(\boldsymbol{x}_q)]$, where $N_{\text{T}} = \lceil ker_{\text{T}}/2 \rceil + \Delta$, and $\boldsymbol{H}_{\text{s}}$ is a $(N_{\text{R}}+1)^2 \times (N_{\text{T}}+1)^2$ scattering matrix. Δ is an integer chosen to reduce the truncation error to an acceptable level, for example $\Delta = 3$ gives an error less than 3%.

We name $\boldsymbol{J}_{\text{R}}$ as the *receiver sampling matrix* and $\boldsymbol{J}_{\text{T}}$ as the *transmitter sampling matrix* since the receiver and transmit antenna positions and orientation information are contained within these matrices. These two matrices are known and fixed for a given receiver/transmitter antenna structure. They characterize the effect of finite separation of antennas on the overall channel matrix $\boldsymbol{H}$.

The scattering gain function $A(\widehat{\boldsymbol{\phi}}, \widehat{\boldsymbol{\varphi}})$ can be expressed using spatial basis functions on scattering free spheres of the receivers and transmitters. A natural choice is to use spherical harmonics, thus

$$A(\widehat{\boldsymbol{\phi}}, \widehat{\boldsymbol{\varphi}}) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n}\sum_{n'=0}^{\infty}\sum_{m'=-n'}^{n'} \beta_{nm}^{n'm'} Y_{nm}(\widehat{\boldsymbol{\phi}})\overline{Y_{n'm'}}(\widehat{\boldsymbol{\varphi}}). \quad (26)$$

It can be shown that the $\beta_{nm}^{n'm'}$ is the $(n+1)^2 - n + m, (n'+1)^2 - n' + m$ element of the scattering matrix $\boldsymbol{H}_{\text{s}}$. Since spherical harmonics are orthonormal to each other we have

$$\beta_{nm}^{n'm'} = \int_{\Omega}\int_{\Omega} A(\widehat{\boldsymbol{\phi}}, \widehat{\boldsymbol{\varphi}})\,\overline{Y_{nm}}(\widehat{\boldsymbol{\phi}})Y_{n'm'}(\widehat{\boldsymbol{\varphi}})\,d\Omega(\widehat{\boldsymbol{\phi}})\,d\Omega(\widehat{\boldsymbol{\varphi}}), \quad (27)$$

The scattering matrix $\boldsymbol{H}_{\text{s}}$ is generally unknown and for a random scattering environment, $\beta_{nm}^{n'm'}$ are random variables. In the wireless literature on MIMO systems, the elements of $\boldsymbol{H}$ are modelled as random variables. However in our model, fixed portions due to transmit and receiver antenna configurations are factored out leaving only the parameters of the scattering environment to be modelled as random variables or to be determined using channel estimation techniques. This is a clear advantage of the current model over existing models since one can explicitly see the role of antenna configuration within the model.

C. Line array

In MIMO literature uniformly spaced linear arrays (ULAs) has been used extensively for spatial channel modelling (e.g., [7], [8]) and capacity calculations. Here we show that the model developed in previous section is valid even for non-uniformly spaced linear arrays. In fact, our model has a simplified structure for this special case.

Without lost of generality aligned the transmit linear array and receiver array to z-axis of respective co-ordinate systems. Thus, $\boldsymbol{x}_q = (x_q, 0, 0)$ and $\boldsymbol{z}_q = (z_p, 0, 0)$ where x_q and z_p are the distances to the qth transmitter and pth receiver from the transmitter origin and receiver origin respectively. Now, the receiver configuration matrix reduces to

$$\boldsymbol{J}_{\mathrm{R}} = \begin{pmatrix} \mathcal{J}_0(\boldsymbol{z}_1) & \cdots & \mathcal{J}_{N_{\mathrm{R}}}(\boldsymbol{z}_1) \\ \vdots & \ldots & \vdots \\ \mathcal{J}_0(\boldsymbol{z}_P) & \cdots & \mathcal{J}_{N_{\mathrm{R}}}(\boldsymbol{z}_P) \end{pmatrix}, \tag{28}$$

where

$$\mathcal{J}_n(\boldsymbol{x}) \triangleq \mathcal{J}_{n0}(\boldsymbol{x}) \tag{29}$$

and similarly the transmitter configuration matrix is given by

$$\boldsymbol{J}_{\mathrm{T}} = \begin{pmatrix} \mathcal{J}_0(\boldsymbol{x}_1) & \cdots & \mathcal{J}_{N_{\mathrm{T}}}(\boldsymbol{x}_1) \\ \vdots & \ldots & \vdots \\ \mathcal{J}_0(\boldsymbol{x}_Q) & \cdots & \mathcal{J}_{N_{\mathrm{T}}}(\boldsymbol{x}_Q) \end{pmatrix}. \tag{30}$$

Now the scattering matrix $\boldsymbol{H}_s$ is $(N_{\mathrm{R}} + 1) \times (N_{\mathrm{T}} + 1)$ and given by

$$\boldsymbol{H}_{\mathrm{s}} = \begin{pmatrix} \beta_0^0 & \cdots & \beta_0^{N_{\mathrm{T}}} \\ \vdots & \ddots & \vdots \\ \beta_{N_{\mathrm{R}}}^0 & \cdots & \beta_{N_{\mathrm{R}}}^{N_{\mathrm{T}}} \end{pmatrix} \tag{31}$$

where

$$\beta_n^{n'} \triangleq \beta_{n0}^{n'0} \tag{32}$$

$$= \int_0^{\pi} \int_0^{\pi} A(\theta_{\mathrm{T}}; \theta_{\mathrm{R}}) Y_{n0}(\theta_{\mathrm{T}}) \overline{Y_{n'0}}(\theta_{\mathrm{R}}) d\theta_{\mathrm{T}} d\theta_{\mathrm{R}} \tag{33}$$

and

$$A(\theta_{\mathrm{T}}; \theta_{\mathrm{R}}) \triangleq \int_0^{2\pi} \int_0^{2\pi} A(\widehat{\boldsymbol{\phi}}, \widehat{\boldsymbol{\varphi}}) d\phi_{\mathrm{T}} d\phi_{\mathrm{R}} \tag{34}$$

$$= \sum_{n=0}^{\infty} \sum_{n=0}^{\infty} \beta_n^{n'} \overline{Y_{n0}}(\theta_{\mathrm{T}}) Y_{n'0}(\theta_{\mathrm{R}}) \tag{35}$$

D. Degrees of Freedom in Spatially Constrained Arrays

It is well known that the rank of the channel matrix $\boldsymbol{H}$ gives the effective number of independent parallel channels between the transmit and receive antenna arrays, and thus determines the degrees of freedom in the system. For the decomposition (24) we have $\text{rank}\{\boldsymbol{H}\} = \min\{\text{rank}\{\boldsymbol{J}_{\mathrm{T}}\}; \text{rank}\{\boldsymbol{J}_{\mathrm{R}}\}; \text{rank}\{\boldsymbol{H}_{\mathrm{s}}\}\}$, which, for a large number of antennas within a constrained region of space with *rich scattering* [10], becomes $\min\{(N_{\mathrm{R}} + 1)^2, (N_{\mathrm{T}} + 1)^2\}$ and $\min\{(N_{\mathrm{R}} + 1), (N_{\mathrm{T}} + 1)\}$ for an arbitrary 3D array configuration and a line array respectively. Therefore we see that the number of modes for the scatter-free transmit and receive regions limit the degrees of freedom of constrained arrays.

IV. Applications

We believe that the development in this paper has wide range of applications. In this section we briefly discuss some of the possible applications.

There has been larger number of papers showing the capacity gain of MIMO systems (e.g., [2]). Recent work has shown the limitation of MIMO systems due to channel correlation [3], [4]. Using channel decomposition developed in this paper, we can further analyze the MIMO capacity in terms of physical antenna configuration.

Basic spatial diversity systems assumed uncorrelated signals at each diversity branch. This assumption is invalid in most practical systems where finite antenna separation and non isotropic scattering cause correlated signals. With the current channel model, we can redevelop spatial diversity systems taking into account the effects of antenna separation.

The model can also be used to design new spatial multiplexing systems or improve existing systems taking into consideration of deterministic portions of the channel matrix. Recall that the BLAST [2] and most of the space-time coding systems [11], [12] assumed uncorrelated received signals. Performance of such systems under correlated channel conditions are yet to be seen in the literature and currently we are studying these systems with the decomposition introduced in this paper.

V. Conclusions

The channel matrix of a MIMO system can be factored into fixed (and known) and random matrices where the deterministic portion depends on receiver and transmitter antenna configurations. The spherical harmonic decomposition of wavefields has used to propose a novel 3D spatial channel model.

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