New Results on the Capacity of M-ary PPM Ultra- WideBand Systems

J. Zhang, R. A. Kennedy, T. D. Abhayapala
Department of Telecommunications Engineering, RSISE
The Australian National University, Canberra ACT 0200, Australia
Email: jian@syseng.anu.edu.au

***Abstract*— In this paper, some new results on the capacity of a typical M-ary pulse position modulation (M-PPM) time hopping (TH) Ultra- Wideband (UWB) system are presented. Previous results using a "pure PPM model" are proven to exaggerate the real capacity of a UWB system. Based on an extended model containing correlator and soft decision decoding, the capacity is evaluated in the single-user case and in the case of a system with asynchronous multiple user interference (MUI) when the inputs are equiprobable. It is found that only when bit- signal-to-noise ratio (bit-SNR) is high enough, larger M leads to higher capacity; and for a specific M, the optimal values of PPM time offset parameter T_d, which maximize the capacity, are independent of bit-SNR. The influence of MUI on capacity is detrimental, especially in the case of high bit-SNR.**

I. Introduction

Ultra- Wideband (UWB) uses extremely narrow (short) pulses to convey information so energy spreads over a bandwidth as wide as several GigaHertz. With the celebrated Shannon theory [1], the capacity can be written as $C = W\log_2(1+\text{SNR})$ bits/s, where W is the bandwidth. It appears that a UWB system can provide enormous capacity.

M-ary Pulse Position Modulation (M-PPM) and Time Hopping (TH) are widely adopted in UWB systems today. For such systems with discrete inputs, the above capacity formula, which is only applicable to channels with continuous inputs and outputs, is no longer strictly applicable. Starting from considerations of mutual information theory, the capacity of PPM UWB systems can be developed from first principles.

PPM has been used in wireless infrared and optical communications for a long time, and the capacity of a PPM system has been well-founded based on a so-called "pure PPM model", that is, an AWGN channel model with power-constrained discrete M-PPM inputs and unconstrained continuous outputs [2]. Capacity analysis of UWB systems can appeal to these results to make an estimation of capacity as in [3]. However, the pure PPM model is overly simplistic and unrealistic to reflect the precise structure of a UWB system. Actually, as we will see in section II-B, the results based on the pure PPM model exaggerate the capacity of a UWB system.

To bound the capacity of a typical M-PPM TH UWB system more tightly, new models have to be constructed to reflect all critical components of a UWB system particularly the correlator receiver and soft decision decoding. A general method to evaluate the capacity, based on a model containing detailed receiver information, is given in [4], that is, to construct a discrete memoryless system with channel transition probabilities obtained from the symbol error rate produced by the detection algorithm. However, this method requires high computational complexity. In this paper, we introduce an alternative way to evaluate the capacity. The general expression of capacity is derived without approximation. The *unshaped capacity* (UC), which is defined as the capacity achieved when the inputs are equiprobable, is investigated in detail in the single-user case and in the case of a UWB system with asynchronous multiple user interference (MUI). Finally, numerical results which corroborate the theory are given.

II. System Model and Capacity Theory

In a typical M-ary PPM TH UWB system, M correlators are used to generate M decision variables, the maximum corresponds to the best estimate of the symbol transmitted and soft decision decoding is applied.

A. Signal and UWB System Model

In an asynchronous multiple-user system, without loss of generality, user 1 and the first transmitted symbol are supposed to be desired. When N_u users are active, for an AWGN channel without multipath, the signal entering each of user 1's M correlators can be represented as

$$r(t) = \sum_{k=1}^{N_u} A_k s^{(k)}(t-\tau_k) + n(t) \tag{1}$$

with

$$s^{(k)}(t) = \begin{cases} \sum_{j=0}^{N_s-1} \omega(t - jT_f - c_j^{(1)}T_c - dT_d), & \text{when } k = 1, \\ \sum_{j=-\infty}^{\infty} \omega(t - jT_f - c_j^{(k)}T_c - d_j^{(k)}T_d), & \text{when } k \geq 2, \end{cases} \tag{2}$$

where A_k is the attenuation of user k's signal over the channel, and τ_k is the time asynchronism between user 1 and user k. The noise process $n(t)$ is assumed to be AWGN with mean zero, variance σ^2 and independent of $s^{(k)}(t)$. The waveform $\omega(t)$ represents the received monocycle pulse. The element of TH sequences, $c_j^{(k)}$, is a random integer in the range $1 \leq c_j^{(k)} \leq N_c$ and T_c is the TH chip width. The modulated data d and $d_j^{(k)}$ are both integers in the range $[1, M]$ and T_d is the time offset of M-ary PPM. The T_f is the frame period, then

0-7803-7802-4/03/$17.00 © 2003 IEEE

$T_b = N_s T_f$ is the bit period, where N_s can be regarded as the spread gain. These parameters are carefully designed to avoid overlapping of pulses belonging to different frames, that is,

$$N_c T_c + M T_d < T_f - T_\omega, \tag{3}$$

where T_ω is the pulse width. Defining the template signal in user 1's m^{th} correlator as

$$X_{1,m} = \sum_{j=0}^{N_s-1} \omega(t - jT_f - c_j^{(1)} T_c - mT_d - \tau_1), \tag{4}$$

and applying soft-decision decoding, the output of m^{th} correlator becomes

$$y_m = u_m + n_{N_u,m} + n_m, \tag{5}$$

where

$$\begin{aligned} u_m &= \sum_{j=0}^{N_s-1} \int_{\tau_1+jT_f}^{\tau_1+(j+1)T_f} A_1 s^{(1)}(t-\tau_1) X_{1,m}\, dt \\ &= A_1 N_s \rho((d-m)T_d) \triangleq \phi((d-m)T_d), \end{aligned} \tag{6}$$

$$n_{N_u,m} = \sum_{j=0}^{N_s-1} \int_{\tau_1+jT_f}^{\tau_1+(j+1)T_f} \sum_{k=2}^{N_u} A_k s^{(k)}(t-\tau_k) X_{1,m} dt, \tag{7}$$

$$n_m = \sum_{j=0}^{N_s-1} \int_{\tau_1+jT_f}^{\tau_1+(j+1)T_f} n(t) X_{1,m}\, dt, \tag{8}$$

where the constant $m \in [1, M]$, the random variable $d \in [1, M]$ whose i^{th} sample is assumed to be i, and $\rho(\tau) = \int_{-\infty}^{\infty} \omega(t-\tau)\omega(t)dt / \int_{-\infty}^{\infty} \omega(t)\omega(t)dt$ represents the normalized autocorrelation of $\omega(t)$, i.e., $\rho(0) = 1$.

B. UWB Channel Model for Capacity Analysis

The basic channel model of an M-ary PPM TH UWB system for capacity analysis can be simplified as a cascaded channel as shown in Fig. 1(a) in which S, Z and Y correspond to $A_1 s^{(1)}(t)$, $r(t)$ and y_m respectively, and Channel 1 is AWGN corresponding to $n(t)$. The capacity based on a pure PPM model, as studied in [3], is actually the capacity of the part from S to Z.

Since Y can be totally determined by Z and Z only depends on S, the cascaded channel in Fig. 1(a) forms a Markov chain. We then have the following relation for average mutual information which is also known as *data processing inequality*

$$I(S; Z) = I(S; Y) + I(S; Z|Y) \geq I(S; Y) \tag{9}$$

With equality if and only if, S and Z are statistically independent conditional on Y [4], [5]. While obviously, S and Z are correlated due to $Z = S + N$ where N is a Gaussian process, and the transformation from Z to Y is irreversible although the correlator operation is deterministic. Then S and Z are mutually dependent even given Y, and $I(S; Z) > I(S; Y)$. According to the relationship between the capacity and average mutual information, we conclude that the capacity based on the pure PPM model is larger than the real capacity of UWB systems.

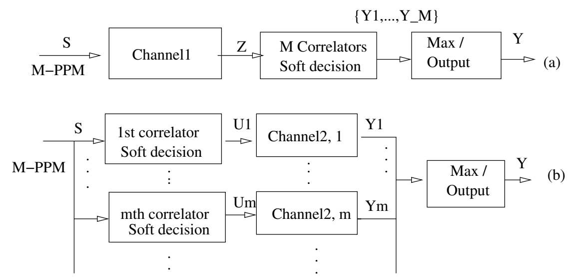

Fig. 1. UWB channel original (a) and equivalent (b) models for capacity analysis.

As the evaluation of $I(S; Z|Y)$ is not straightforward, we endeavor to create a new model where only discrete time AWGN is between inputs and outputs. Following an idea in [6], we rearrange the order of correlator and channel to obtain an equivalent model as shown in Fig. 1(b). Let $Y_m = U_m + V_m$, where the discrete random variable U_m, corresponding to u_m in (6), represents the output of m^{th} correlator, and the continuous Gaussian variable V_m (channel 2,m in Fig. 1(b)), corresponding to $n_{N_u,m} + n_m$ in (5), represents an Gaussian random variable independent of U_m as will be shown later.

Now, we derive the capacity expression for the UWB system based on the model in Fig. 1(b). For the m^{th} path, we have

$$\begin{aligned} I(Y_m; S, U_m) &= I(S; Y_m) + I(U_m; Y_m|S) \\ &= I(U_m; Y_m) + I(S; Y_m|U_m). \end{aligned}$$

Because Y_m can be totally determined by U_m in the new equivalent model, we get

$$p(y_m|u_{m,\ell}) = p(y_m|u_{m,\ell}, s_i) \Rightarrow I(S; Y_m|U_m) = 0, \tag{10}$$

where s_i, $u_{m,\ell}$, and y_m are the samples of S, U_m and Y_m respectively, and $p(\cdot)$ represents probability mass function (pmf) for discrete variables or probability density function (pdf) for continuous variables.

Since the autocorrelation $\rho(\cdot)$ is an even-function and so is $\phi(\cdot)$ according to (6), the transformation from s to u_m is a many-to-one mapping. Once s_i ($d_i = i$) is given, $u_{m,\hat{i}}$ is determined by (6). Then

$$p(u_{m,\ell}|s_i) = \begin{cases} 1 & \text{if } \ell = \hat{i} \\ 0 & \text{if } \ell \neq \hat{i}, \end{cases}$$

Together with (10), some further results can be derived as

$$\begin{aligned} p(y_m, u_{m,\ell}|s_i) &= p(y_m|u_{m,\ell}, s_i) p(u_{m,\ell}|s_i) \\ &= \begin{cases} p(y_m|u_{m,\hat{i}}) & \text{if } \ell = \hat{i} \\ 0 & \text{if } \ell \neq \hat{i}, \end{cases} \\ p(u_{m,\ell}|s_i) p(y_m|s_i) &= p(u_{m,\ell}|s_i) \sum_{\ell} p(y_m, u_{m,\ell}|s_i) \\ &= \begin{cases} p(y_m|u_{m,\hat{i}}) & \text{if } \ell = \hat{i}, \\ 0 & \text{if } \ell \neq \hat{i}. \end{cases} \end{aligned}$$

Thus

$$\begin{aligned} &p(u_{m,\ell}|s_i) p(y_m|s_i) = p(y_m, u_{m,\ell}|s_i) \\ &\Rightarrow I(U_m; Y_m|S) = 0 \Rightarrow I(S; Y_m) = I(U_m; Y_m). \end{aligned}$$

So the capacity has the expression of

$$\begin{aligned} C &= \max_{p(s)} I(S;Y) \\ &= \max_{m} \max_{p(u_{m,\ell})} \{H(Y_m) - H(Y_m|U_m)\} \\ &= \max_{m}\{ \max_{p(u_{m,\ell})} H(Y_m) - H(V_m)\}, \end{aligned} \quad (11)$$

where m stands for m^{th} path containing m^{th} correlator, the system capacity corresponds to the path with maximum average mutual information.[1] The last equality comes from a well known fact that for an AWGN channel, the conditional entropy of the output given the input, $H(Y_m|U_m)$, is equal to the noise entropy $H(V_m)$ and thus independent of the input distribution U_m if U_m and V_m are statistically independent [4], [5]. And $H(V_m)$ is given by

$$H(V_m) = 1/2 \ln(2\pi e \sigma_v^2) \quad (12)$$

where σ_v^2 is the variance of V_m.

For convenience, the function $\ln(\cdot)$ instead of $\log_2(\cdot)$ is used in the computation of entropies in this paper. So the unit of all related capacities is nats/symbol unless stated otherwise. To transfer to the unit of bits/symbol, just divide the value by $\ln 2$.

III. Capacity Analysis

The self-entropy $H(Y_m)$ is given by

$$H(Y_m) = -\int_{-\infty}^{\infty} p(y_m) \ln p(y_m)\ dy_m, \quad (13)$$

where

$$\begin{aligned} p(y_m) &= \sum_{\ell} p(u_{m,\ell}) p(y_m|u_{m,\ell}) \\ &= \sum_{\ell} p(u_{m,\ell}) p(v_m)|_{v_m = y_m - u_{m,\ell}}. \end{aligned} \quad (14)$$

As $\phi(\cdot)$ is an even-function, $u_{m,\ell}$ may correspond to s_i and $s_{i'}$ which results to $p(u_{m,\ell}) = p(\phi((i-m)T_d)) + p(\phi((m-i')T_d))$ where $i - m = m - i'$. However, it can be verified that the value of $p(y_m)$ in (14) will not be affected by the use of $p(\phi((i-m)T_d))$ and $p(\phi((m-i')T_d))$ separately instead of combining them, neither will the capacity. So we can simplify $u_{m,\ell}$ and $p(u_{m,\ell})$ as

$$\begin{aligned} & u_{m,i} = \phi((i-m)T_d), \\ & p(u_{m,i}) = p(s_i)|_{s_i = \phi^{-1}(u_{m,i})}, \\ & i \in [1, M],\ i - m \in [1-m, M-m]. \end{aligned} \quad (15)$$

where $\phi^{-1}(\cdot)$ represents the inverse function of $\phi(\cdot)$.

Although Y_m is a continuous variable and the maximum entropy of a continuous random variable is achieved when it is Gaussian distributed, Y_m can not be Gaussian distributed since U_m is a discrete variable. When $M = 2$, $H(Y_m)$ is maximized when the inputs $u_{m,i}$ are equally probable since the transition probabilities $p(y_m|u_{m,i})$ exhibit a form of symmetry. However, this is not the solution for $M > 2$. As verified in simulation, equiprobable inputs do not maximize $H(Y_m)$ for $M > 2$. As a matter of fact, it is hard to find the optimal input probability to achieve the capacity. We instead focus on the so-called capacity of a UWB system when inputs are equally probable, that is, $p(s_i) = 1/M$, and then $p(u_{m,i}) = 1/M$. We name this capacity as *Unshaped Capacity* (UC) and represent it with $\widetilde{C}$.

[1] This is a reasonable approximation according to the definition of the Capacity.

A. Unshaped Capacity of a Single-user UWB System

In the single-user case, $n_{N_u,m} = 0$ and $V_m = n_m$. As the template signal $X_{1,m}$ is deterministic, n_m is a Gaussian random variable [7] with mean zero and variance $N_s\rho(0)\sigma^2 = N_s\sigma^2 \triangleq \sigma_1^2$. It is obvious that for all m, V_m has the same distribution. We know that the deterministic functions of two variables are also independent if the variables are independent [8]. According to the assumption of independence between $n(t)$ and $s^{(1)}(t)$, V_m and U_m are independent. So the UC of a single user UWB system becomes

$$\widetilde{C}_1 = \max_{m} H(Y_m) - 1/2 \ln(2\pi e \sigma_1^2), \quad (16)$$

where $H(Y_m)$ has the form of (13) with

$$p(y_m) = \frac{1}{\sqrt{2\pi} M \sigma_1} \sum_{i=1}^{M} \exp\Big[-\frac{\big(y_m - u_{m,i}\big)^2}{2\sigma_1^2}\Big]. \quad (17)$$

Let $x_m = y_m/\sigma_1$, then

$$\begin{aligned} p(x_m) &= \sigma_1 p_{y_m}(\sigma_1 x_m) \\ &= \frac{1}{\sqrt{2\pi} M} \sum_{i=1}^{M} \exp\Big[-\frac{1}{2}\big(x_m - \frac{u_{m,i}}{\sigma_1}\big)^2\Big], \end{aligned} \quad (18)$$

and (16) can be rewritten as

$$\begin{aligned} \widetilde{C}_1 &= \max_{m}\{-\ln\sqrt{2\pi e} - \int_{-\infty}^{\infty} p(x_m) \ln p(x_m) dx_m\} \\ &= \max_{m}\{-\ln\sqrt{2\pi e} - E_{x_m}[\ln p(x_m)]\}, \end{aligned} \quad (19)$$

where $E_{x_m}[\ln p(x_m)]$ represents the expectation of $\ln p(x_m)$. It implies that $\widetilde{C}_1$ depends on the ratio of

$$\frac{u_{m,i}}{\sigma_1} = \frac{A_1 N_s \rho((i-m)T_d)}{\sqrt{N_s}\sigma} = \sqrt{\mathrm{SNR_b}}\ \rho((i-m)T_d), \quad (20)$$

rather than on either separately. The bit- signal-to-noise ratio (bit-SNR) $\mathrm{SNR_b}$ is equal to $A_1^2 N_s \rho(0)/\sigma^2 = A_1^2 N_s/\sigma^2$.

Exact computation of $\widetilde{C}_1$ is extremely complex. An approximate computation using numerical techniques, such as Riemann Integral or Gaussian-Hermite quadrature [9], computes the integral in the expression of $\widetilde{C}_1$. An alternate method is to estimate the expectation in (19) via Monte Carlo simulation. The simulation process is first to generate pseudo-random numbers according to the pdf $p(x_m)$, then apply the function of $\ln(p(x_m))$ to every sample to produce new data. Finally their mean is the expectation needed. Both methods lead to similar results. Some results obtained from Riemann integral are shown in section IV.

B. Unshaped Capacity of a UWB System with MUI

The capacity of multi-user systems are always evaluated through a parallel channel model like that in [4]. Here we focus on another aspect, the influence of asynchronous MUI on the UC of a UWB system.

Similar to the widely used assumption in signal detection, we regard the MUI as Gaussian distributed noise. Then the sample of V_m becomes $v_m = n_m + n_{N_u,m}$, where $n_{N_u,m}$ has Gaussian distribution $N(m_u, \sigma_u^2)$. Due to the independence of n_m and $n_{N_u,m}$, V_m is Gaussian distributed with mean m_u and variance $\sigma_1^2 + \sigma_u^2 \triangleq \sigma_2^2$.

The evaluation of m_u and σ_u^2 is similar to the process in [10]. The key idea to obtain a simple and closed-form expression is to assume an ideal but reasonable distribution of τ_k. Under the further constraint of $N_cT_c + MT_d < T_f/2 - T_m$, we obtain $m_u = 0$ and

$$\sigma_u^2 = N_s \sigma_a^2 \sum_{k=2}^{N_u} A_k^2, \tag{21}$$

where

$$\sigma_a^2 = T_f^{-1} \int_{-\infty}^{\infty} \Big[\int_{-\infty}^{\infty} \omega(x-s)\omega(x)dx \Big]^2 ds. \tag{22}$$

It should be stated that the results above do not depend on the template signal $X_{1,m}$ and a specific correlator, so the distributions of $n_{N_u,m}$ of all correlators have the same form. Actually, $X_{1,m}$ does not contribute to the derivation directly. Both results of (21) and (22) can be obtained once the expectation and variance conditional on a random async-time-delay variable $\alpha_{1,k}(u)$ [10] are evaluated.

According to the assumption that the signals transmitted by all users are mutually independent, V_m and U_m are independent. Then the UC of a UWB system with asynchronous MUI becomes

$$\widetilde{C}_2 = \max_m \{H(Y_m) - 1/2 \ln(2\pi e \sigma_2^2)\}, \tag{23}$$

where $H(Y_m)$ has the form of (13) with

$$p(y_m) = \frac{1}{\sqrt{2\pi} M \sigma_2} \sum_{i=1}^{M} \exp\Big[-\frac{(y_m - u_{m,i})^2}{2\sigma_2^2} \Big]. \tag{24}$$

It can be verified that all results from (18) to (20) are applicable in this part, after replacing σ_1 by σ_2.

IV. Numerical Results

A typical received monocycle $\omega(t/\tau_c)$ at the output of the antenna subsystem can be represented as

$$\omega(t/\tau_c) = [1 - 4\pi(t/\tau_c)^2] \exp[-2\pi(t/\tau_c)^2], \tag{25}$$

where τ_c parameterizes the effective pulse width. Then the normalized autocorrelation function of $\omega(t/\tau_c)$ is

$$\rho(\frac{\tau}{\tau_c}) = [1 - 4\pi(\frac{\tau}{\tau_c})^2 + \frac{4\pi^2}{3}(\frac{\tau}{\tau_c})^4] \exp[-\pi(\frac{\tau}{\tau_c})^2]. \tag{26}$$

To make the analysis not explicitly depend on τ_c, we use the ratio of t/τ_c as the variable.

It is obvious that $\widetilde{C}_1$ and $\widetilde{C}_2$ also depend on the time offset T_d. If differentiating $\widetilde{C}_1$ in (19) (or $\widetilde{C}_2$) with respect to T_d/τ_c, we can find that

$$\frac{\partial \ \rho((i-m)T_d/\tau_c)}{\partial \ (T_d/\tau_c)} = 0 \tag{27}$$

is one of the solutions to make $(\partial C_1)/(\partial(T_d/\tau_c)) = 0$. This equation is applicable to any pulse when the corresponded autocorrelation function ρ is applied. Substituting (26) for $\rho((i-m)T_d/\tau_c)$, we obtain $(i-m)T_d/\tau_c = \pm 0.5408$ and ± 1.1398. Note that it means when one of these values is assigned, the corresponding value of $\widetilde{C}_1$ could be a local extremum or a global extremum. Obviously, the extremum is independent of the bit-SNR. This is the situation shown in Fig. 2, in which the UC of a binary PPM UWB system is demonstrated. With $M = 2$, the extremum is achieved when $T_d/\tau_c = 0.5408$ and 1.1398. We can find that $\widetilde{C}_1$ has the maximum when $T_d/\tau_c = 0.5408$ and the minimum when $T_d/\tau_c = 1.1398$ for all bit-SNRs. When the bit-SNR is large enough, the capacity increases quickly and reaches the maximum while $(T_d/\tau_c) < 0.5408$.

The UC of a M-PPM UWB system for various bit-SNR is illustrated in Fig. 3, where optimal values of T_d/τ_c are chosen to maximize the UC for every M and bit-SNR. As comparisons, results obtained via the method in [3] are also shown. In experiments, we find the optimal values for fixed M but varied bit-SNR are similar; while with M increasing, the values decrease. Some optimal values of T_d/τ_c are shown in Table I. We can see that these values do approximately match the points of $(i-m)T_d/\tau_c = \pm 0.5408$.

Fig. 3 implies that larger M need not lead to higher UC. Only when the bit-SNR is large enough, higher UC is achieved for larger M than smaller one. This is a surprising result, compared to those in previous research such as [3] and [2], in which equiprobable inputs are claimed to achieve the general capacity and capacity with larger M is always higher than that with smaller M for every bit-SNR. A possible reason for the difference is that in our research, the values of input symbols

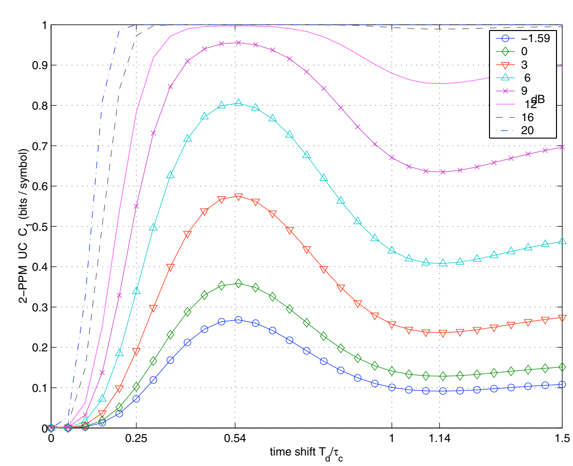

Fig. 2. Binary PPM UC versus time shift T_d/τ_c for various bit-SNR in the single-user case.

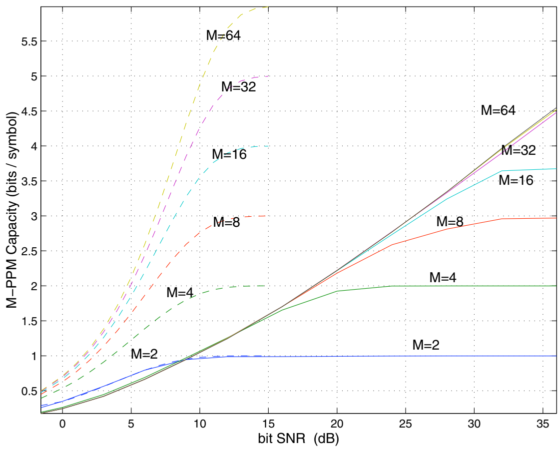

Fig. 3. Capacity of M-ary PPM UWB system for various bit-SNR in the single-user case. Solid line: $\tilde{C}_1$ in (16); Dashed line: results from [3].

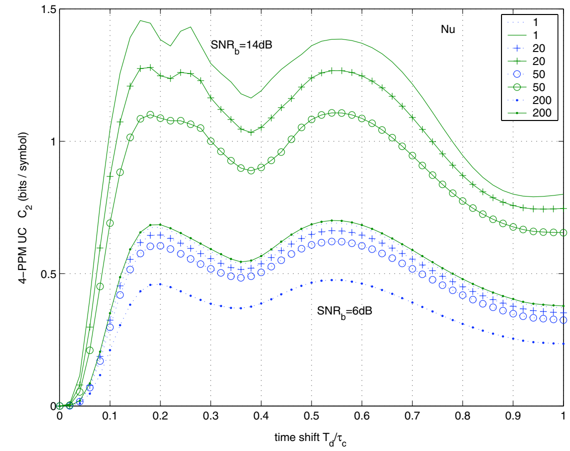

Fig. 4. 4-PPM UC versus time shift T_d/τ_c for bit-SNR of 14dB (solid lines) and 6dB (dotted lines) for multiuser (N_u=1, 20, 50, 200) case.

involved in capacity computation are not those in $[1, M]$ but the sample points of autocorrelation function dependent on specific monocycle. From the signal-constellation viewpoint, the minimal distance between symbols is not the invariant value of unity but a value that varies with M for an optimal T_d/τ_c. When M increases, the distance decreases. Then higher bit-SNR is required to combat noise and to output resolvable decision variables. According to this reason, the phenomena observed here are independent of the shape of pulses.

Through the analysis in Section III-B, the UC of UWB systems with MUI can be simulated and computed in the same way as the single-user case. For the waveform of (25), $\sigma_a^2 = 0.5156/T_f$ can be obtained from (22). Assuming the system is well power-controlled, that is, $A_k = A_1, k \in [2, N_u]$, then $\sigma_2^2 = 0.5156 N_s (N_u - 1) A_1^2/T_f + \sigma_1^2$, and

$$\frac{u_{m,i}}{\sigma_2} = \frac{\rho((i-m)T_d)}{\sqrt{0.5156(N_u-1)/T_b + \mathrm{SNR}_b^{-1}}}. \qquad (28)$$

A 4-PPM system is exemplified in Fig. 4 to demonstrate how the UC is affected by MUI, where N_u=1, 20, 50 and 200, and $T_b = 500\tau_c$. From Fig. 4, the influence of MUI on the UC is severe, especially for large bit-SNR. According to (28), for a specific user number N_u, increasing the bit period T_b can lessen the noise variance σ_2 and mitigate the influence. However, T_b also directly contributes to the bits/second capacity. This is a tradeoff to consider when the bits/second capacity is desired.

TABLE I

THE OPTIMAL VALUES OF T_d/τ_c FOR VARIED BIT-SNR IN A TYPICAL M-PPM UWB SYSTEM USING THE PULSE IN (25).

M	2	4	8	16
T_d/τ_c	0.54	0.17	0.105	0.05
M	32	64	128	
T_d/τ_c	0.024	0.012	0.006	

V. CONCLUSIONS

Previous results from the literature using a pure PPM model are proven to exaggerate the real capacity of a UWB system. Based on an extended model containing correlator and soft decision decoding, we have derived an expression for the capacity and investigated the *Unshaped Capacity* (UC) (the capacity when the UWB system has M-PPM equal probability inputs). It is found that, systems with smaller M can have higher UC when the bit-SNR is small; and only when the bit-SNR is large enough, larger M leads to higher values. It is also found that, for specific M, the optimal values of PPM time-offset T_d that maximize the UC are largely independent of the bit-SNR; while with M increasing, the values decrease and have been analytically characterized. Finally, MUI severely reduces the capacity, especially for high bit-SNR. The model and method here can be extended to multipath channels to broaden the practical usefulness of the results.

REFERENCES

[1] C. E. Shannon, "A mathematical theory of communication," *The Bell System Technical Journal*, vol. 27, pp. 379–423, 623–656, 1948.

[2] S. Dolinar, D. Divsalar, J. hamkins, and F. Pollara, "Capacity of pulse-position modulation (PPM) on gaussian and webb channels," *LPL TMO Progress Report*, pp. 42–142, August 15 2000.

[3] L. Zhao and A. M. Haimovich, "Capacity of M-ary PPM Ultra-Wideband communications over AWGN channels," *IEEE VTC '01 Fall*, pp. 1191–1195, Oct. 2001.

[4] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: John Wiley & Sons Inc., 1991.

[5] R. G. Gallager, *Information Theory And Reliable Communication*. John Wiley & Sons, Inc., 1968.

[6] S. Zerisberg, C. Műller, and J. Siemes, "Performance limits of Ultra-Wideband basic modulation principles," *IEEE International Conf. on Communications (ICC)*, pp. 816–820, 2001.

[7] S. Verdú, *Multiuser Detection*. Cambridge University Press, 1998.

[8] V. K. Rohatgi and A. K. M. E. Saleh, *An Introduction to Probability and Statistics*, 2nd ed. New York: John Wiley & Sons, Inc., 2001.

[9] P. F. Davis and P. .Rabinowitz, *Methods of Numerical Integration*, 2nd ed. San Diego: Academic Press, Inc., 1984.

[10] M. Z. Win and R. A. Scholtz, "Ultra-wide bandwidth time-hopping spread-spectrum impulse radio for wireless multiple-access communications," *IEEE Trans. Commun.*, vol. 48(4), pp. 679–691, Apr. 2000.