Antenna Saturation Effects on MIMO Capacity

T. S. Pollock, T. D. Abhayapala, and R. A. Kennedy
National ICT Australia
Research School of Information Sciences and Engineering
The Australian National University, ACT 0200, Australia
Email: tony.pollock@anu.edu.au

***Abstract*— A theoretically derived antenna saturation point is shown to exist for MIMO systems, at which the system suffers a capacity growth decrease from linear to logarithmic with increasing antenna numbers. We show this saturation point increases linearly with the radius of the region containing the receiver antennas and is independent of the number of antennas. Using an alternative formulation of capacity for MIMO systems we derive a closed form capacity expression which uses the physics of signal propagation combined with statistics of the scattering environment. This expression gives the capacity of a MIMO system in terms of antenna placement and scattering environment and shows that the saturation effect is due to spatial correlation between receiver antennas.**

I. Introduction

Multiple-Input Multiple-Output (MIMO) communication systems using multi-antenna arrays simultaneously during transmission and reception have generated significant interest in recent years. Theoretical work of [1] and [2] showed the potential for significant capacity increases in wireless channels utilizing spatial diversity. However, in reality the capacity is significantly reduced when the signals received by different antennas are correlated, corresponding to a reduction of the effective number of sub-channels between transmit and receive antennas [2], [3]. Previous studies have given insights and bounds into the effects of correlated channels [3]–[5], however most have been for a limited set of channel realizations.

In contrast, one contribution of this paper is an alternative form for capacity which overcomes current limitations, that is, with additional theory for modelling scattering environments which we refine here, we derive a model which can be readily reconciled with a multitude of scattering models and allows us to derive a closed form capacity expression. Using this new model we show that there is a only a small subset of the eigenvalues generated by a spatial correlation matrix between elements of an array which effect capacity, regardless of the number of antennas in the array. This result leads to the channel capacity suffering a saturation effect with increasing antenna numbers, after which additional antennas give limited capacity growth.

Although previous work has focused on eigenvalues as a means to bound or simplify the tedious ergodic capacity computation [1], [5]–[7], the emphasis was on the eigenvalues of the random channel matrix product giving little insight into environmental effects on capacity. In this paper we expose characteristics of the eigenvalues of an antenna spatial correlation matrix that leads to an antenna saturation effect, which has the detrimental effect of reducing capacity growth from linear to logarithmic with increasing antenna numbers.

II. Capacity of Multiple Antenna Systems

Consider a MIMO system consisting of S transmitting antennas with statistically independent equal power components each with Gaussian distributed signals, and Q receiving antennas. Then for a fixed linear channel, represented by a $S \times Q$ channel matrix $\boldsymbol{H}$, with additive white Gaussian noise the channel capacity is given by [1], [2]

$$C_{\boldsymbol{H}} = \log \left| \boldsymbol{I}_Q + \frac{\eta}{S} \boldsymbol{H}\boldsymbol{H}^\dagger \right| \tag{1}$$

where η is the average signal-to-noise ratio (SNR) at each receiver branch, $|\cdot|$ is the determinant operator, and $\dagger$ the Hermitian operator. In the case of a random channel model the channel matrix is stochastic hence the capacity given by (1) is also random. In this situation the mean (ergodic) capacity is obtained by taking the expectation of capacity $C_{\boldsymbol{H}}$ over all possible channel realizations,

$$\widetilde{C} = E\left\{ \log \left| \boldsymbol{I}_Q + \frac{\eta}{S} \boldsymbol{H}\boldsymbol{H}^\dagger \right| \right\}. \tag{2}$$

Equation (2) is often used in Monte Carlo simulations to provide capacity results for different channel matrix models, however these simulations offer little insight and fail to provide a rigorous demonstration into factors determining capacity. Some analytical lower and upper bounds on the ergodic capacity have been derived (e.g., see [2], [3]), more recently, [4] gave an upper bound on ergodic capacity based on the correlation between each channel matrix element. In contrast, we use a capacity formula based on the spatial correlation of the antennas at the receiver and the statistics of the scattering environment, and thus remove any explicit dependencies on the random channel matrix. Due to subtle differences between our capacity result and that of the classical formulation we provide a detailed derivation. To study the effects of correlation on capacity we restrict ourselves to receive correlation only and assume no transmit correlation, a valid assumption given the less geometrical size restrictions for base-station arrays.

A. Alternative MIMO Capacity Derivation

Let $\boldsymbol{x} = [x_1, x_2, \ldots, x_S]^T$ be the vector of symbols sent by the S transmitting antennas, $\boldsymbol{n} = [n_1, n_2, \ldots, n_Q]^T$ be the zero mean additive white gaussian noise vector, and

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$\boldsymbol{y} = [y_1, y_2, \ldots, y_Q]^T$ be the vector of received symbols, where T denotes the vector transpose, then

$$\boldsymbol{y} = \boldsymbol{r} + \boldsymbol{n} \tag{3}$$

where $\boldsymbol{r}$ is the vector of Q noiseless symbols received after propagation of S symbols $\boldsymbol{x}$ through a flat fading channel. The channel capacity of a MIMO channel with total transmit power restricted to P is defined as [8]

$$C = \max_{p(\boldsymbol{x}):\mathrm{tr}(\boldsymbol{V_x})\leq P} I(\boldsymbol{x};\boldsymbol{y}) \tag{4}$$

where $p(\boldsymbol{x})$ is the transmitter statistical distribution, $\boldsymbol{V_x} = E\left\{\boldsymbol{x}\boldsymbol{x}^\dagger\right\}$ is the covariance matrix of the transmitted symbols $\boldsymbol{x}$, with $E\{\cdot\}$ the expectation operator, and $I(\boldsymbol{x};\boldsymbol{y})$ is the mutual information of the transmitted and received symbols. Assuming the receiver noise is independent from the transmitted symbols, the mutual information is given by

$$I(\boldsymbol{x};\boldsymbol{y}) = \mathcal{H}(\boldsymbol{y}) - \mathcal{H}(\boldsymbol{n}) \tag{5}$$

with $\mathcal{H}(\cdot)$ the differential entropy of a continuous random variable, hence maximizing capacity is achieved by maximizing the entropy $\mathcal{H}(\boldsymbol{y})$. The entropy of a complex gaussian vector $\boldsymbol{x}$ has the following inequality [1]

$$\mathcal{H}(\boldsymbol{x}) \leq \log|\pi e \boldsymbol{V_x}| \tag{6}$$

with log base 2 and entropy is expressed in bits. Equality in (6) is achieved if and only if $\boldsymbol{x}$ is a circularly symmetric complex Gaussian. It is easy to show if $\boldsymbol{x}$ is a circularly symmetric complex Gaussian then so is $\boldsymbol{y}$, thus, assuming optimal gaussian distribution for the transmit vector $\boldsymbol{x}$, the mutual information becomes

$$I(\boldsymbol{x};\boldsymbol{y}) = \log\frac{|\boldsymbol{V_y}|}{|\boldsymbol{V_n}|}$$

where $\boldsymbol{V_y} = E\{\boldsymbol{y}\boldsymbol{y}^\dagger\}$, and $\boldsymbol{V_n} = E\{\boldsymbol{n}\boldsymbol{n}^\dagger\}$ are the received and noise covariance matrices, respectively. Let $\boldsymbol{V_r} = E\{\boldsymbol{r}\boldsymbol{r}^\dagger\}$ be the covariance matrix of the noiseless received symbols, then, assuming the receiver noise is independent from the transmitted signals, the received covariance matrix $\boldsymbol{V_y}$ becomes

$$\boldsymbol{V_y} = \boldsymbol{V_r} + \boldsymbol{V_n} \tag{7}$$

then, assuming uncorrelated noise in each receiver branch, the noise covariance matrix $\boldsymbol{V_n}$ becomes $\sigma^2\boldsymbol{I}_Q$, with noise variance σ^2, and the mutual information can be written as,

$$I(\boldsymbol{x};\boldsymbol{y}) = \log\left|\boldsymbol{I}_Q + \frac{1}{\sigma^2}\boldsymbol{V_r}\right|. \tag{8}$$

In the situation where the transmitter has no knowledge about the channel, it is optimal to uniformly distribute the power across all the transmit antennas [1], thus from (4) and (8) the MIMO channel capacity is given by

$$C = \log\left|\boldsymbol{I}_Q + \frac{1}{\sigma^2}\boldsymbol{V_r}\right|. \tag{9}$$

The capacity in (9) is the Shannon capacity for the noiseless received symbol correlation matrix $\boldsymbol{V_r}$ which is defined by the statistical scattering environment, antenna numbers and placement.

In this paper we use a receiver antenna spatial correlation approach which gives the capacity without the explicit need for the use of a random channel matrix. Here the scattering environment and antenna placement is captured by the noiseless received symbols correlation matrix $\boldsymbol{V_r}$ and is utilized to give an analytical capacity formula.

B. Spatial Correlation Matrix Approach

Define the normalized spatial correlation between the complex envelopes of the received signal at two antennas p and q as

$$\rho_{pq} \triangleq \frac{E\{r_p r_q^*\}}{\sigma_s^2} \tag{10}$$

where σ_s^2 is the average signal power received at any receive antenna, assuming normalized channel gains. We can then write

$$\frac{1}{\sigma^2}\boldsymbol{V_r} = \eta\boldsymbol{R}_Q \tag{11}$$

with the $Q \times Q$ spatial correlation matrix

$$\boldsymbol{R}_Q \triangleq \begin{bmatrix} \rho_{11} & \cdots & \rho_{1Q} \\ \vdots & \ddots & \vdots \\ \rho_{Q1} & \cdots & \rho_{QQ} \end{bmatrix} \tag{12}$$

where each ρ_{pq} depends on antenna separation and the power distribution of the scattering environment.

For a two dimensional propagation environment [9] showed that

$$\rho_{pq} = \sum_{n=-\infty}^{\infty} i^n \beta_n J_n(k\|\boldsymbol{z}_p - \boldsymbol{z}_q\|)e^{in\phi_{pq}} \tag{13}$$

where $\boldsymbol{z}_p$ is the location of the pth point, ϕ_{pq} is the angle of the vector connecting $\boldsymbol{z}_p$ and $\boldsymbol{z}_q$, $k = 2\pi/\lambda$ is the wavenumber with λ the wavelength, and J_n is the nth order bessel function of the first kind. The coefficients β_n characterize any possible scattering environment and are given by

$$\beta_n = \int_0^{2\pi} \mathcal{P}(\varphi)e^{-in\varphi}\mathrm{d}\varphi \tag{14}$$

with $\mathcal{P}(\varphi)$ the average power density distribution over φ the angle to the scatters. For essentially all common choices of $P(\varphi)$: von-Mises, gaussian, truncated gaussian, uniform, piecewise constant, polynomial, Laplacian, Fourier series expansion, etc., there is a closed form expression for the β_n [9]. Therefore we have a closed form representation for the spatial correlation (13) and, as we will see next, for the capacity of a MIMO channel.

The capacity in (9) can now be expressed as

$$C = \log|\boldsymbol{I}_Q + \eta\boldsymbol{R}_Q| \tag{15}$$

which is the capacity for the MIMO system given the scattering environment, described by the average power density distribution $\mathcal{P}(\varphi)$, and antenna placement. The capacity given by (15) is maximized when there is no correlation between the receive antennas, i.e., $\boldsymbol{R}_Q = \boldsymbol{I}_Q$, giving,

$$C_{\max} = Q \log(1+\eta). \tag{16}$$

Therefore, in the idealistic situation of zero correlation between receiver antennas we see the best capacity growth achievable is linear in the number of antennas. This result agrees with the traditional capacity formulation [1], [2] which is widely used to advocate the use of MIMO systems. However, as we shall show in the following section, the linear capacity growth result does not hold for the more realistic situation where the antennas are restricted to a region in space.

III. Capacity of a Uniform Circular Array in a 2D Isotropic Diffuse Field

Consider a uniform circular array (UCA) with radius r and Q receiver elements. Denote the set $\{d_\ell\}, \ell = 0, \dots, Q-1$ as the distance between any element and the other $Q-1$ elements in the array (in a clockwise or anticlockwise direction), with $d_0 = 0$ being the distance between the element and itself, then

$$d_\ell \triangleq 2r \sin(\pi\ell/Q). \tag{17}$$

For the special case of scattering over all angles in the plane we have a 2D isotropic diffuse field (often referred to as a *rich* scattering environment) at the receiver and the normalized average power reduces to $\mathcal{P}(\varphi) = 1/2\pi$, $\varphi \in [0, 2\pi)$, giving the spatial correlation between any two points in the plane as

$$\rho_{pq} = J_0(k\|\boldsymbol{z}_p - \boldsymbol{z}_q\|). \tag{18}$$

Define the spatial correlation between any element on the UCA and its ℓth neighbour as

$$\rho_\ell \triangleq J_0(k\, d_\ell) \tag{19}$$

then due to UCA symmetry, $\rho_\ell = \rho_{Q-\ell}$, and the correlation matrix becomes a $Q \times Q$ symmetric circulant matrix,

$$\boldsymbol{R}_Q = \mathrm{Circ}\left[\rho_0, \rho_1, \dots, \rho_{\lceil \frac{Q-1}{2} \rceil}, \rho_{\lfloor \frac{Q-1}{2} \rfloor}, \dots, \rho_2, \rho_1\right] \tag{20}$$

where $\lceil\cdot\rceil$ and $\lfloor\cdot\rfloor$ are the ceiling and floor operators respectively, and

$$\mathrm{Circ}\left[x_1, x_2, \dots, x_N\right] \triangleq \begin{bmatrix} x_1 & x_2 & \cdots & x_N \\ x_N & x_1 & \cdots & x_{N-2} \\ \vdots & & \ddots & \vdots \\ x_2 & x_3 & \cdots & x_1 \end{bmatrix} \tag{21}$$

defines the circulant matrix.

The determinant in the capacity formula (15) can be expanded by the product of eigenvalues of the argument, giving,

$$C = \sum_{m=0}^{Q-1} \log(1+\eta\lambda_m) \tag{22}$$

where $\lambda_m \in \sigma(\boldsymbol{R}_Q)$ are the Q eigenvalues of the circulant spatial correlation matrix $\boldsymbol{R}_Q$. Therefore we see that the capacity is governed by the eigenvalues of the spatial correlation matrix, and as such their properties dictate the behavior of the capacity given differing scattering environments, antenna numbers and placement. It is important to note that the eigenvalues in (22) belong to the spatial correlation matrix $\boldsymbol{R}_Q$ and not the random channel matrix product $\boldsymbol{H}\boldsymbol{H}^\dagger$, as analyzed in other work (for example see [1], [5]–[7]). Unlike previous work where the eigenvalues of the channel matrix product are used simply as a means to bound or simplify the tedious ergodic capacity computation, in the following we expose characteristics of the spatial correlation matrix eigenvalues that lead to valuable insights into the channel capacity.

A. Eigenvalues of Spatial Correlation Matrix $\boldsymbol{R}_Q$

The eigenvalues of the symmetric circulant matrix $\boldsymbol{R}_Q$ are given by a simple closed form expression [10]

$$\lambda_m = \sum_{\ell=0}^{Q-1} \rho_\ell e^{i2\pi m\ell/Q}. \tag{23}$$

For a UCA in a 2D isotropic diffuse field the correlation coefficients are real and symmetric, hence (23) represents the Discrete Cosine Transform (DCT) of the spatial correlation coefficients;

$$\lambda_m = \sum_{\ell=0}^{Q-1} \rho_\ell \cos\left(2\pi m\ell/Q\right). \tag{24}$$

Since $\boldsymbol{R}_Q$ is a positive-semidefinite Hermitian matrix and with the properties of the DCT it is easy to show the following:

$$\lambda_m \in \mathbb{R} \tag{25}$$

$$\lambda_m \geq 0 \tag{26}$$

$$\lambda_{Q-m} = \lambda_m\,,\ m > 0 \tag{27}$$

that is, all the eigenvalues of $\boldsymbol{R}_Q$ are real, non-negative and symmetric.

Theorem 1 (Maximum Eigenvalue Threshold): For a UCA of radius r in a 2D isotropic diffuse field define the Maximum Eigenvalue Threshold:

$$M \triangleq \lceil \pi e r/\lambda \rceil \tag{28}$$

then, for $m \in \{M+1, Q-M-1\}$ the eigenvalues λ_m are vanishingly small.

Before proving Theorem 1 we clarify its significance with the following interpretation:

For any UCA in a 2D isotropic diffuse field there is a finite set of non-zero spatial correlation matrix eigenvalues, where the set size increases linearly with the radius of the array and is independent of the number of antennas.

Proof: Substitution of (19) and (17) into (24) gives

$$\lambda_m = \sum_{\ell=0}^{Q-1} J_0(2kr\sin\left(\pi\ell/Q\right))\cos\left(2m\pi\ell/Q\right) \tag{29}$$

letting $\xi = \pi\ell/Q$ and assuming a large number of antennas, we can approximate (29) with the integral

$$\lambda_m \approx \frac{Q}{\pi}\int_0^\pi J_0(2kr\sin\xi)\cos\left(2m\xi\right)\mathrm{d}\xi \tag{30}$$

for $m = \{0, 1, \ldots, \lceil (Q-1)/2 \rceil\}$. Using the identity [11, p.32]

$$J_n^2(z) = \frac{1}{\pi} \int_0^{\pi} J_0(2z \sin\psi) \cos(2n\psi) \mathrm{d}\psi \tag{31}$$

then the eigenvalues can be expressed as

$$\lambda_m \approx Q J_m^2(kr) \tag{32}$$

which is asymptotically equal to (29) with the antenna number.

Using the the following bound [12, p.362] on the bessel functions for $n \geq 0$

$$|J_n(z)| \leq \frac{|z|^n}{2^n \Gamma(n+1)} \tag{33}$$

the eigenvalues are then upper-bounded by

$$\lambda_m \leq Q \left(\frac{(C/2\lambda)^m}{\Gamma(m+1)} \right)^2 \tag{34}$$

with $C = 2\pi r$ the circumference of the circular array. Since Gamma function $\Gamma(m+1)$ increases faster than the exponential $(C/2\lambda)^m$ then (34) will rapidly approach 0 for some $m > 0$ for which $\Gamma(m+1) > (C/2\lambda)^m$. Using a relaxed Stirling lower bound[1] for $\Gamma(m+1)$, we wish to find m for which

$$\left(\frac{m}{e}\right)^m > \left(\frac{C}{2\lambda}\right)^m. \tag{35}$$

The inequality in (35) is clearly satisfied when $m > Ce/2\lambda$, asserting that m must be an integer and from the definition of C we see that the eigenvalues vanish for $m > \lceil \pi e r/\lambda \rceil$, thus giving the maximum eigenvalue threshold in (28).

Given the symmetric nature of the eigenvalues in (27) then for any number of antennas, $Q \geq 2M+1$, there is a finite set of $2M+1$ non-zero eigenvalues,

$$\boldsymbol{\lambda} = \{\lambda_0, \lambda_1, \cdots, \lambda_M, \lambda_{Q-M}, \cdots, \lambda_{Q-2}, \lambda_{Q-1}\} \tag{36}$$

whose number of elements grows only with the radius of the array, and is independent on the number of antennas. ■

Fig. 1 shows the eigenvalues of the spatial correlation matrix $\boldsymbol{R}_Q$ for various UCA radii in a 2D isotropic diffuse field. Shown as a solid black line, it can be seen that the theoretical maximum eigenvalue threshold derived in Theorem 1 gives a good indication of the maximum non-vanishing eigenvalue for each radius.

B. Capacity Growth Limits: Antenna Saturation

Due to the dependence of the capacity formula (22) on the eigenvalues of the spatial correlation matrix we see that Theorem 1 has significant implications on the capacity growth with increasing antenna numbers. In this section we show that this fixed set size of eigenvalues, regardless of the number of antennas, leads to an antenna saturation effect on MIMO capacity.

Theorem 2 (Antenna Saturation Point): For a UCA of radius r in a 2D isotropic diffuse field define a saturation point

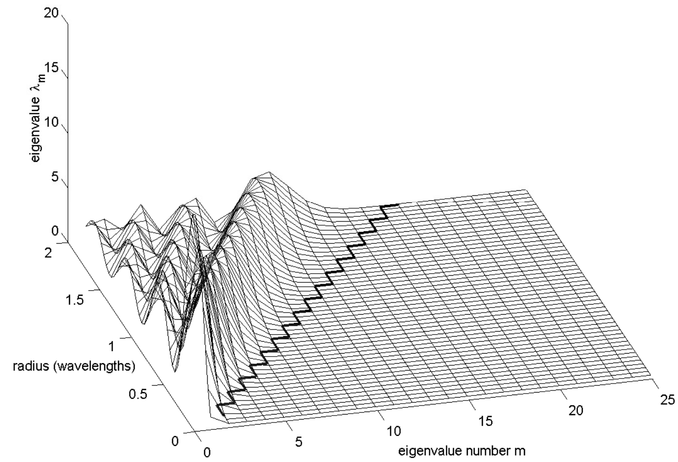

Fig. 1. The eigenvalues of the spatial correlation matrix for various UCA radii in a 2D isotropic diffuse scattering field. The dark solid line represents the theoretical Maximum Eigenvalue Threshold derived in Theorem 1, and clearly shows the boundary between the significant and vanishing eigenvalues of the spatial correlation matrix for each array radius.

Q_M as the minimum number of antennas that generate a full set of non-zero eigenvalues λ_m;

$$Q_M \triangleq 2M + 1 \tag{37}$$

then, for any $Q \geq Q_M$ the channel capacity is governed by

$$C \approx \log(1 + \widehat{\eta}\lambda_0) + 2 \sum_{m=1}^{M} \log(1 + \widehat{\eta}\lambda_m) \tag{38}$$

where $\widehat{\eta} = (Q/Q_M)\eta$ is the scaled average SNR ratio at each receiver branch.

Before giving a proof of Theorem 2 we give the following interpretation:

For a MIMO system with a UCA in a 2D isotropic diffuse field there exists a saturation point in the number of antennas, which is dependent only on the radius of the array, after which the addition of more antennas gives diminishing (logarithmic) capacity gains.

Proof: Denoting $\lambda_m^{(Q)} \in \sigma(\boldsymbol{R}_Q)$ as the eigenvalues generated from Q antennas on a UCA of radius r, then the capacity (22) can be expressed as

$$\sum_{m=0}^{Q-1} \log\left(1 + \eta\lambda_m^{(Q)}\right) \tag{39}$$

which, when using (27) and assuming an odd number of antennas can be written as[2]

$$C = \log\left(1 + \eta\lambda_0^{(Q)}\right) + 2 \sum_{m=1}^{(Q-1)/2} \log\left(1 + \eta\lambda_m^{(Q)}\right). \tag{40}$$

Consider the UCA placed in a 2D isotropic diffuse field, then as a direct result of Theorem 1 for $Q \geq 2M+1$ the channel

[1] $\Gamma(z+1) > \sqrt{2\pi z}\, z^z e^{-z} > z^z e^{-z}$, $z > 0$

[2] from Theorem 1 the case of even Q gives identical results, however to simplify notation we assume an odd number of antennas

capacity given by (40) is well approximated using the set of $2M+1$ non-vanishing eigenvalues, that is,

$$C \approx \log\left(1+\eta\lambda_0^{(Q)}\right) + 2\sum_{m=1}^{M}\log\left(1+\eta\lambda_m^{(Q)}\right). \quad (41)$$

Given two UCAs of equal radius r with antenna numbers Q_1, $Q_2 \geq 2M+1$, and spatial correlation matrix eigenvalues $\lambda_m^{(Q_1)}$ and $\lambda_m^{(Q_2)}$ respectively, then from (32) we have the following relationship between eigenvalues for systems with different numbers of receive antennas,

$$\frac{\lambda_m^{(Q_1)}}{Q_1} \approx \frac{\lambda_m^{(Q_2)}}{Q_2} \quad (42)$$

with the approximation asymptotically equal with the number of antennas. Define $Q_M \triangleq 2M+1$ as the minimum number of antennas which generate the full set of non-zero eigenvalues, then letting $Q_1 = Q_M$ and $Q_2 = Q$ we have

$$\lambda_m^{(Q)} \approx \frac{Q}{Q_M}\lambda_m \quad (43)$$

where λ_m are the eigenvalues of the spatial correlation matrix $\boldsymbol{R}_{Q_M}$. Thus the eigenvalues for any UCA of radius r with number of antennas $Q \geq Q_M$ are simply scaled versions of the eigenvalues generated by an array with Q_M antennas. Substituting (43) into (41) gives

$$C \approx \log\left(1+\frac{\eta Q\lambda_0}{Q_M}\right) + 2\sum_{m=1}^{M}\log\left(1+\frac{\eta Q\lambda_m}{Q_M}\right) \quad (44)$$

which behaves logarithmically with Q since the eigenvalues are constant for all Q, hence the capacity gain is reduced to logarithmic growth once the antenna number reaches the saturation point given by Q_M. Let $\widehat{\eta} = (Q/Q_M)\eta$ be the scaled average SNR at each antenna, then the capacity (44) becomes (38), thus we see that the effect of any additional antennas above the saturation point is just an increase in the average SNR, or in other words, a noise-averaging effect due to the assumption of independent noise at each antenna. ■

It can be observed from Fig. 2 that the capacity does indeed increase approximately linearly up until the theoretical saturation point defined in Theorem 2, after which the capacity reduces to logarithmic increase with antenna number. This result has significant implications for practical MIMO systems, we have shown that there is a saturation point in antenna numbers after which there are diminishing returns in capacity for additional antennas, therefore the saturation point gives the optimal number of antennas that achieve maximum capacity with minimum cost. Further to the UCA case, empirical studies have shown that there are only ever $2M+1$ significant eigenvalues generated by arbitrarily placed antennas within a circular region of radius r. We believe the saturation effect seen in UCAs also holds for any antenna configuration within a circular region and we are currently developing theoretical results to support this.

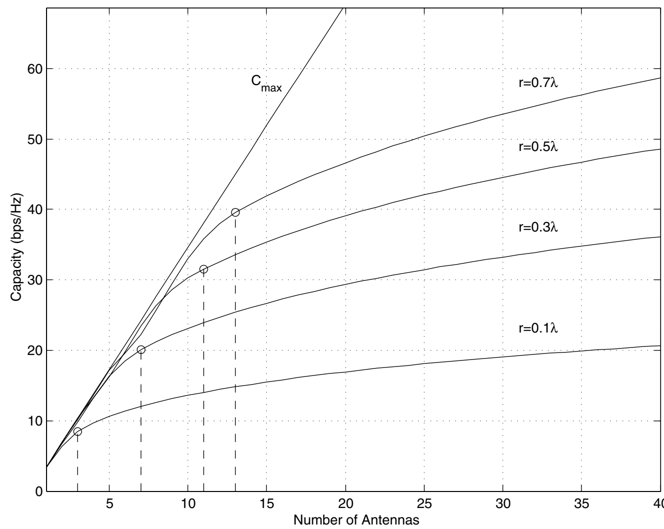

Fig. 2. Capacity of MIMO systems for various antenna numbers of a UCA with radii $r = 0.1, 0.3, 0.5$, and 0.7 wavelengths in a 2D isotropic diffuse scattering field, along with the theoretical maximum capacity. As indicated by the dashed lines for each radii, the Antenna Saturation Point theoretically derived in Theorem 2 gives a good indication where the MIMO system saturates and hence increasing antenna numbers gives only marginal capacity gain.

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