Performance of Ultra- Wideband Correlator Receiver Using Gaussian Monocycles

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***Abstract*— This paper investigates the performance of Ultra-Wideband (UWB) correlator receivers for Gaussian monocycles under the condition of equal mean power and provides constructive reference to the selection of pulses. Several channel situations are examined including ideal single user AWGN channel, non-ideal synchronous, multipath fading and multiple access interference. Both numerical and analytical techniques show that the shape of pulses have notable impact on the performance of correlator receivers, especially on the interference resistance ability and signal-to-noise ratio (SNR) of the output. The results are also extended to the field of fractional bandwidth to better understand the possible relationship between fractional bandwidth and correlator receivers.**

I. Introduction

Ultra- Wideband (UWB) or impulse radio for commercial communication application is a recent innovation. M-ary pulse position modulated (PPM) time-hopping (TH) UWB systems are considered to be the most competitive techniques for indoor WLAN or WPAN communications [1] [2]. TH-UWB can be regarded as some kind of CDMA technique. The main difference between UWB and conventional CDMA systems, exists in the so-called shaping pulse.

Typically, extremely narrow (short) pulses are modulated and emitted to convey information in a UWB system. These pulses are also referred to as shaping waveforms, but they are quite different from those used in conventional CDMA systems like DS-CDMA. In a DS-CDMA system, the shaping waveform (chip waveform) is primarily used to spread the spectrum, limit the bandwidth of the output and separate a group of channels [3]. While in a UWB system, the pulse plays a key role and primarily determines the performance of the system. First of all, due to the pulse's characteristic of ultra wide bandwidth and excellent time-resolution, UWB systems can provide very large capacity. Then, signal's power spectrum and the interference to other systems are determined by the shape of the pulse, together with spread spectrum pseudo random (PN) sequences. Last but not least, the pulse contributes to the output of the correlator receiver, and directly affects the performance of the receiver. Basically, it results from the signal's low duty cycle and PPM-TH modulation method, and is exhibited by the autocorrelation function. This influence is almost ignored in previous research. For a DS-CDMA system, it is mainly the correlation of PN codes, but not the shaping pulse, that determines the performance.

Due to their effective simultaneous high time and frequency resolution, Gaussian monocycles are the most widely used pulses in Ultra- Wideband (UWB) systems at present. Gaussian monocycles [4] are a class of pulses obtained by taking successive derivatives of the basic Gaussian waveform. In this paper, in order to discover the effect of pulses' shape on receivers, the performance of correlator receiver is evaluated for Gaussian monocycles in various channel conditions. Some numerical results are illustrated, which indicate that the shape of pulses has a significant impact on the performance of the receivers. For the rigor of conclusions, the analytical description is given after the numerical results. Since the pulse shape is associated with fractional bandwidth (FB) [5], which is fundamental to a UWB system, and defined as $2(f_H - f_L)/(f_H + f_L)$, where f_H and f_L are the frequencies measured at the -10dB emission points, the problem whether FB is related to the performance of receivers is important. This problem is discussed in section IV.

II. System model

For simplicity, binary PPM is adopted and the results can be readily extended to M-ary PPM system.

A. Gaussian Monocycles

The basic Gaussian waveform has the form of

$$\omega_0(t) = e^{-2\pi(\frac{t}{t_p})^2}, \tag{1}$$

and its n-order derivative named as n-order monocycle is

$$\omega_n(t) = \varepsilon_n \frac{d^n}{dt^n}(e^{-2\pi(\frac{t}{t_p})^2}), \tag{2}$$

where t_p parameterizes the effective width of the pulse and ε_n is introduced to normalize the energy of the pulses $\omega_n(t)$ such that for the autocorrelation function

$$\rho_n(\tau) \triangleq \int_{-\infty}^{\infty} \omega_n(t)\omega_n(t-\tau)\, dt = (-1)^n \frac{d^{(2n)}\ \rho_0(\tau)}{d\ \tau^{2n}}, \tag{3}$$

$\rho_n(0) = 1$ for all n. It means the transmitted signal consisted of a specific monocycle $\omega_n(t)$ has equal mean power for every n. The point which minimizes $\rho_n(\delta)$ is defined as

$$\delta_{n,\text{opt}} \triangleq \arg_\delta \min \rho_n(\delta). \tag{4}$$

In this paper, one value of t_p is set to 0.7531ns to be consistent with previous literature such as [6]. Another of

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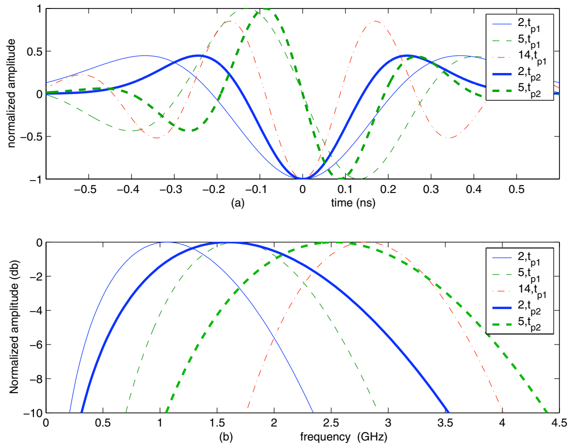

Fig. 1. (a) Time domain waveforms, (b) frequency spectrum of n-order Gaussian monocycles, where $t_{p1} = 0.7521$ns, $n = 2, 5, 14$; $t_{p2} = 0.5$ns, $n = 2, 5$.

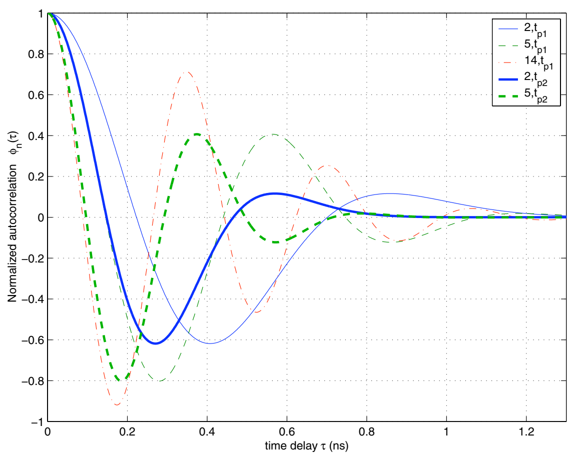

Fig. 2. Normalized autocorrelation of n-order Gaussian monocycles, where $t_{p1} = 0.7521$ns, $n = 2, 5, 14$; $t_{p2} = 0.5$ns, $n = 2, 5$.

0.5ns is used as a comparison. Fig. 1 shows the time domain waveform and frequency spectrum of some monocycles. Note that all monocycles' -10dB FB are larger than 20% which satisfies the definition of a UWB system. Fig. 2 shows their autocorrelations. We define the segment from $\tau = 0$ to the minimum point $\delta_{n,\text{opt}}$ as the *mainlobe* of the autocorrelation.

Since it does not affect the analysis, we will ignore the possible derivative transformation of the waveform during propagation and focus on the received waveform.

For binary PPM TH-UWB system, the k^{th} transmitter's signal conveying i^{th} bit can be represented as

$$\omega_{\text{bit}}^{(k)}(t,i) = \sum_{j=iN_s}^{(i+1)N_s-1} \omega_n(t - jT_f - c_j^{(k)}T_c - \delta d_i^{(k)}), \quad (5)$$

where $c_j^{(k)} \in [1, N_c]$ is the time hopping sequence, T_c is the TH chip width, δ is the time offset of binary PPM, $d_i^{(k)} \in \{0, 1\}$ is the i^{th} bit data to be transmitted and is assumed to be equiprobable. T_f is the frame period, then bit period $T_b = N_sT_f$ where N_s is the number of pulses to represent one bit. The limit $N_cT_c + 2\delta < T_f$ is assumed to avoid ISI interference.

Without lose of generality, we assume that user 1 ($k = 1$) is desired. When N_u users are active, the signal entering user1's receiver corrupted by multipath, can be modeled as

$$r(t) = \sum_{k=1}^{N_u}\sum_{m=1}^{M}\sum_{i=0}^{\infty} A_{km}\omega_{\text{bit}}^{(k)}(t - \tau_k - \lambda_m, i) + n(t), \quad (6)$$

where A_{km} is the amplitude of k^{th} user's m^{th} multipath, λ_m is the multipath time delay, M is the number of multipaths, and τ_k is the time asynchronism between k^{th} transmitter and user1's receiver. The waveform $n(t)$ represents white Gaussian noise with a mean of zero and variance σ^2.

B. Receiver Signal Processing

The system is assumed to be perfectly synchronized ($\tau_1 = 0$) unless stated otherwise. The well-known optimal processor for single user AWGN channel is correlator plus maximum likelihood detection, which corresponds to the following soft decision rule for i^{th} bit

$$\text{decide “}d_i^{(1)} = 0\text{” } \Longleftrightarrow$$

$$\underbrace{\sum_{j=iN_s}^{(i+1)N_s-1} \int_{jT_f}^{(j+1)T_f} r(t)v_n(t - jT_f - c_j^{(1)}T_c)dt}_{\alpha} > 0, \quad (7)$$

where $v_n(t) = \omega_n(t) - \omega_n(t - \delta)$, and α is the decision variable.

When other users are present, this detector is also used as the suboptimum one, while multiple access interference is approximated as white gaussian noise.

III. Performance of UWB Correlator Receiver

As the same waveform is used in all frame periods by all users, each correlation operation, including crosscorrelation, has the form of an autocorrelation operation. So we define the bit correlation function between user k and l as

$$\phi_{kl}(\tau) \triangleq \int_{iN_sT_f}^{((i+1)N_s-1)T_f} \omega_{\text{bit}}^{(k)}(t,i)\omega_{\text{bit}}^{(l)}(t - \tau, i)\ dt, \quad (8)$$

where $\tau \in [-T_f, T_f]$.

A. Ideal Single User AWGN Channel

In this case, (6) becomes

$$r(t) = A_{11}\omega_{\text{bit}}^{(1)}(t,i) + n(t). \quad (9)$$

The signal-to-noise ratio (SNR) in the output of the correlator is [4] [2]

$$\text{SNR}_{\text{out}}^{(1)} = (A_{11}N_sm_p(\delta))^2/\sigma_n^2, \quad (10)$$

where

$$m_p(\delta) = \rho_n(0) - \rho_n(\delta). \quad (11)$$

Let $v_{\text{bit}}^{(1)}(t,i) = \sum_{j=iN_s}^{(i+1)N_s-1} v_n(t - jT_f - C_j^{(1)}T_c)$, then σ_n^2 is the variance of $n_1 \triangleq \int_{iN_sT_f}^{(i+1)N_sT_f-T_f} n(t)v_{\text{bit}}^{(1)}(t,i)\ dt$. Since

$v_{\text{bit}}^{(1)}(t,i)$ is a deterministic signal to the desired receiver, σ_n^2 can be obtained as

$$\begin{aligned}\sigma_n^2 &= E[n_1^2]\\ &= E[n(t)^2]\int_{iN_sT_f}^{(i+1)N_sT_f-T_f} v_{\text{bit}}^{(1)}(t,i)v_{\text{bit}}^{(1)}(t,i)\ dt\\ &= 2\sigma^2[\phi_{11}(0)-\phi_{11}(\delta)]\\ &= 2N_s\sigma^2 m_p(\delta).\end{aligned} \tag{12}$$

Then (10) becomes

$$\text{SNR}_{\text{out}}^{(1)} = \frac{A_{11}^2 N_s m_p(\delta)}{2\sigma^2}, \tag{13}$$

and the probability of bit error is given by

$$P_e = Q\big(\sqrt{\text{SNR}_{\text{out}}^{(1)}}\ \big) = Q\big(\frac{A_{11}\sqrt{N_s m_p(\delta)}}{\sqrt{2}\sigma}\big), \tag{14}$$

where $Q(\cdot)$ is the complementary Gaussian cumulative distribution function.

From the modulation viewpoint, $\delta = \delta_{n,\text{opt}}$ is the optimal choice as the distance is maximized between the two codes. It is also true for a single-user AWGN channel because $\delta_{n,\text{opt}}$ maximizes SNR.

The relationship between $m_p(\delta)$ and δ for some monocycles is shown in Fig. 3 using (11). Given that $\text{SNR}_{\text{out}}^{(1)}$ is directly proportional to $m_p(\delta)$, we find that under equal mean power, monocycles with larger n can provide higher SNR output for small δ, and monocycles with different t_p but same n have the same peak gain. When each $\delta_{n,\text{opt}}$ is selected, as listed in Table I, there is 0.4674dB, 0.7403dB gain between $n = 5, 14$ and $n = 2$ respectively.

B. Non-ideal Synchronization Situation

When a user's receiver and transmitter are not perfectly synchronized, the receiver is confronted with the attenuation of performance. The ability to resist sync-error is very important for such time-stringent systems as TH-UWB.

Supposing the synchronous error is τ_s, then

$$r(t) = A_{11}\omega_{\text{bit}}^{(1)}(t-\tau_s, i) + n(t). \tag{15}$$

Most of the results derived in section III-A remain unchanged except for

$$m_p(\tau_s,\delta) = \rho_n(\tau_s) - \rho_n(\tau_s-\delta). \tag{16}$$

Fig. 4 demonstrates how $m_p(\tau_s,\delta)$ varies with τ_s when $\delta = \delta_{n,\text{opt}}$. It implies that monocycles with smaller n have better sync-error resistance ability than those with larger ones when t_p is fixed. Obviously, this ability depends on mainlobe's width of the signal's autocorrelation.

C. Single User with Multipath Fading

Here we consider a simplified model with only one overlapped multipath and without ISI. The received signal becomes

$$r(t) = A_{11}\omega_{\text{bit}}^{(1)}(t,i) + A_{12}\omega_{\text{bit}}^{(1)}(t-\lambda_2, i) + n(t), \tag{17}$$

where we assume $|A_{12}| \le A_{11}$ and λ_2 is assumed to be a random variable uniformly distributed on $[0, T_m]$ where T_m is the maximum multipath time delay.

According to the decision rule in (7), the probability of bit error is

$$\begin{aligned}P_e &= P[d_i^{(1)}=1]\cdot P[\alpha>0|d_i^{(1)}=1]\\ &\qquad + P[d_i^{(1)}=0]\cdot P[\alpha<0|d_i^{(1)}=0]\\ &= \frac{1}{2}P[A_{11}(\phi_{11}(\delta)-\phi_{11}(0)) + A_{12}\beta + n_1 > 0]\\ &\quad + \frac{1}{2}P[A_{11}(\phi_{11}(\delta)-\phi_{11}(0)) + A_{12}\beta + n_1 < 0]\\ &= \frac{1}{2}Q[\frac{A_{11}Ns(\rho_n(0)-\rho_n(\delta)) - A_{12}\beta}{\sigma_n}]\\ &\quad + \frac{1}{2}Q[\frac{A_{11}Ns(\rho_n(0)-\rho_n(\delta)) + A_{12}\beta}{\sigma_n}]\\ &\le Q[\frac{A_{11}Ns(\rho_n(0)-\rho_n(\delta)) - |A_{12}\beta|}{\sigma_n}]\\ &\triangleq P_{e,\text{upper}},\end{aligned} \tag{18}$$

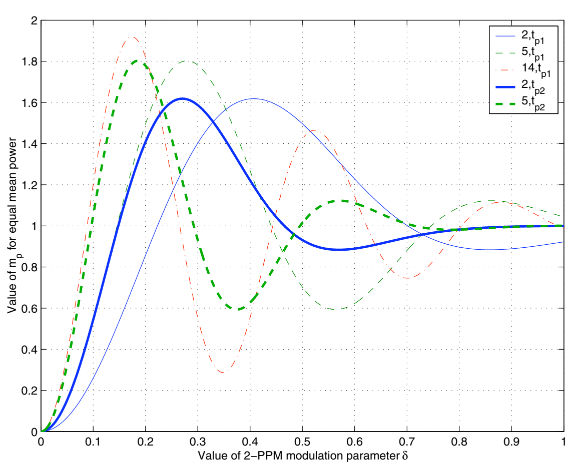

Fig. 3. m_p vs modulation parameter δ for Gaussian monocycles under equal mean power, where $t_{p1} = 0.7521$ns, $n = 2, 5, 14$; $t_{p2} = 0.5$ns, $n = 2, 5$.

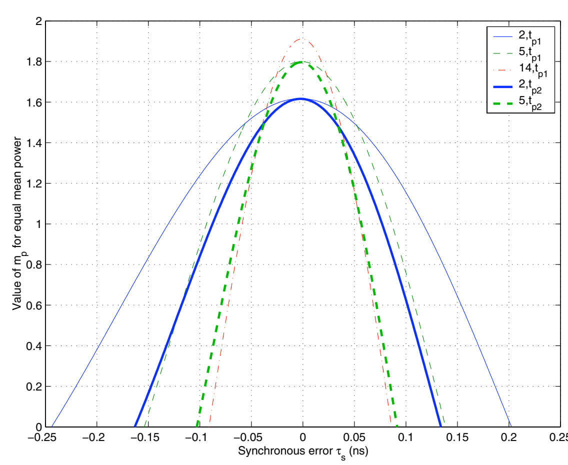

Fig. 4. m_p vs sync-error τ_s for Gaussian monocycles under equal mean power when $\delta = \delta_{n,\text{opt}}$, where $t_{p1} = 0.7521$ns, $n = 2, 5, 14$; $t_{p2} = 0.5$ns, $n = 2, 5$.

where $P[\cdot]$ denotes the probability of an event, and $\beta \triangleq \phi_{11}(\lambda_2) - \phi_{11}(\lambda_2 - \delta))$. When SNR is large, $P_{e,\text{upper}} \approx P_e$.

For a determined δ like $\delta = \delta_{n,\text{opt}}$, the relation between β and λ_2 is similar to that between $m_p(\tau_s, \delta_{n,\text{opt}})$ and τ_s in Fig. 4, and it is regular when λ_2 locates in a short period. However, the regular situation does not remain in a whole frame period and how n and λ_2 affect P_e is not straightforward directly from (18). Since λ_2 is a random variable, it is reasonable to evaluate P_e using the expectation of β ($\bar{\beta}$) instead of β, which is expressed as

$$\bar{\beta} = N_s \Big[\int_{-\infty}^{\infty} E[\omega_n(t-\lambda_2)]\omega_n(t)dt - \int_{-\infty}^{\infty} E[\omega_n(t-\lambda_2)]\omega_n(t-\delta)dt \Big]. \quad (19)$$

The absolute value of $\bar{\beta}$ is demonstrated in Fig. 5 for several values of T_m. Since $Q(.)$ is monotonically decreasing, the larger $\bar{\beta}$ is, the smaller the $P_{e,\text{upper}}$ is. The figure indicates that if t_p is fixed, monocycles with smaller n have smaller $P_{e,\text{upper}}$ for most of the possible values of T_m. This is consistent with [7], which shows how a large fractional bandwidth leads to lower worst-case fading in the presence of multipath. It can also been observed that if n is fixed, monocycles with larger t_p have smaller $P_{e,\text{upper}}$. In other words, monocycles with smaller n and larger t_p, which correspond to signals with broader mainlobes, have better multipath-resistance ability. This is contrary to the performance that narrower pulses have better time resolution ability and multipath-resolving ability which is helpful in RAKE receiver and in the application of localization.

D. Multiple Access Channel

When N_u asynchronous users are active, the SNR of the output of the suboptimal receiver given in [4] [2] is

$$\text{SNR}_{\text{out}}^{(N_u)} = \Big[\frac{1}{\text{SNR}_{\text{out}}^{(1)}} + \frac{1}{N_s} \frac{\sigma_a^2}{m_p^2} \sum_{k=2}^{N_u} \big(\frac{A_{k1}}{A_{11}} \big)^2 \Big]^{-1}, \quad (20)$$

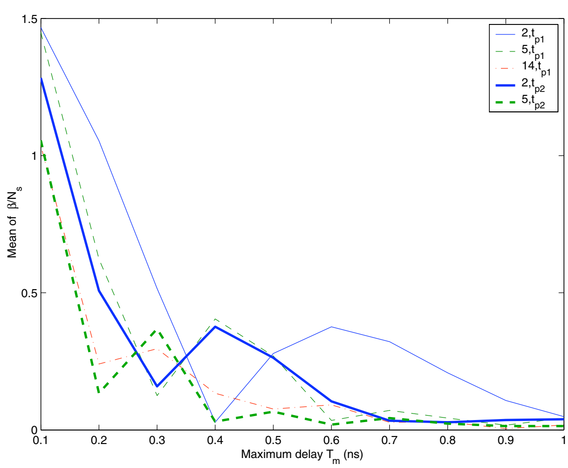

Fig. 5. $|\bar{\beta}/N_s|$ vs T_m for Gaussian monocycles under equal mean power. where $t_{p1} = 0.7521$ns, $n = 2, 5, 14$; $t_{p2} = 0.5$ns, $n = 2, 5$.

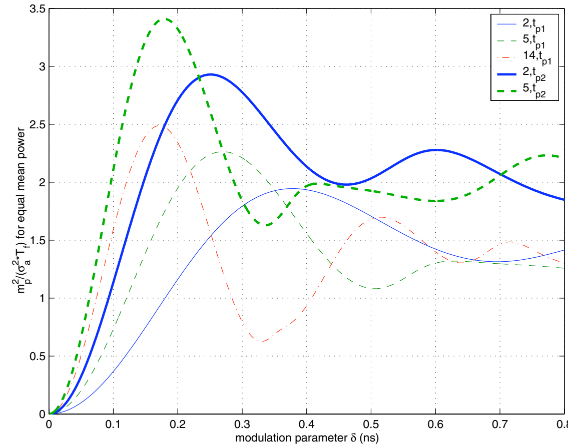

Fig. 6. $m_p^2/(T_f\sigma_a^2)$ vs modulation parameter δ for n-order Gaussian monocycles under equal mean power, where $t_{p1} = 0.7521$ns, $n = 2, 5, 14$; $t_{p2} = 0.5$ns, $n = 2, 5$.

TABLE I

THE OPTIMAL VALUES OF δ FOR m_p OR $(m_p/\sigma_a)^2$. UNIT: *ns*

	$t_p = 0.7531$			$t_p = 0.5$	
n	2	5	14	2	5
$\delta(m_p)$	0.407	0.279	0.174	0.270	0.185
$\delta(\frac{m_p^2}{\sigma_a^2})$	0.378	0.270	0.172	0.251	0.179

where $\text{SNR}_{\text{out}}^{(1)}$ and m_p are from (10) and (11) respectively, and

$$\sigma_a^2 = T_f^{-1} \int_{-\infty}^{\infty} \Big[\int_{-\infty}^{\infty} \omega_n(t-s)v_n(t)dt \Big]^2 ds. \quad (21)$$

The relationship between $m_p^2/(T_f\sigma_a^2)$ and δ is shown in Fig. 6. The values of δ maximizing m_p or $(m_p/\sigma_a)^2$ are given in Table I. From (20), $\text{SNR}_{\text{out}}^{(N_u)}$ can be regarded as a monotonously increasing function of $m_p^2/(T_f\sigma_a^2)$ and $\text{SNR}_{\text{out}}^{(1)}$. Comparing Fig. 6 with Fig. 3, we find that for a specific t_p, $m_p^2/(T_f\sigma_a^2)$ and $\text{SNR}_{\text{out}}^{(1)}$ vary with δ identically. Then it can be concluded that for both single and asynchronous multiuser channels, when t_p is fixed, the monocycles with larger n can provide higher SNR output for small δ including the optimal value in the condition of equal mean power. While surprisingly, monocycles with smaller t_p produce larger gain on SNR in the situation of asynchronous multiple access.

IV. ANALYTICAL DESCRIPTION AND THE ROLE OF FRACTIONAL BANDWIDTH

To show the results clearly, the performance is only illustrated above with the combination of $n = 2, 5, 14$ and $t_p = 0.7531, 0.5$. Actually, the experiments having been done covers much wider range of parameters including $n = 2$ to 20 and multiple values of t_p. All the experiments indicate the same set of regularities with those discussed in section III. Some of those conclusions can also be drawn analytically.

Throughout the analysis above, we find that the receiver's performance is actually connected with signal's autocorrelation, especially the shape of mainlobe. As we know, a signal's autocorrelation and its power spectrum form a Fourier transform-pair. If the width of autocorrelation decreases, the power spectrum will broaden, and vice verse.

For monocycles with fixed t_p, their amplitude spectrums have the form of $10n\log_{10}|(f)| + 10\log_{10}|W_0(f)|$, where $W_0(f)$ is the Fourier transformation of $\omega_0(t)$. Since $10n\log_{10}|(f)|$ is rather flat when f is as large as GHz, these amplitude spectrums are a group of shifted versions of $W_0(f)$ with nearly equal -10dB bandwidth and increased central frequency, as observed in Fig. 1. Again, it can be verified that if function $y(t)$ is the first derivative of function $x(t)$, the autocorrelation of $y(t)$ will be the second derivative of the autocorrelation of $x(t)$. The mainlobe of autocorrelation will become narrow after the derivative operation. As the width of power spectrum containing most energy is almost unchanged, the *sidelobe* of autocorrelation becomes large and fluctuant with high peak and low vale to maintain signal's power inside a certain time domain. This is the situation that we can find in Fig. 2. So the conclusion relating to monocycles with fixed t_p can be straightforwardly extended to all n.

For monocycles with varied t_p but fixed n, with t_p decreasing, the pulse width decreases and the frequency bandwidth increases. The change of autocorrelation is straightforward when we normalize τ with respect to t_p in (3) and obtain $\rho_n(\tau/t_p)$. It is clear that the mainlobe of autocorrelation narrows when t_p decreases and all autocorrelations have same extrema.

The relationship between different t_p and n is ambiguous. If two signals have similar autocorrelation, the one with smaller t_p has a better multiple-user SNR gain and the other with larger n has a better single-user SNR gain. This can direct the selection of pulses used in small or large networks.

Table II lists the fractional bandwidth of some signals. According to the results above, no definite regularity between the performance of correlator receiver and signal's fractional bandwidth is apparent. Now let us rewrite FB as $B/(2f_c)$, where B is frequency bandwidth and f_c is central frequency. Obtainable results are as follows.

- Ristricted to Gaussian monocycles, when we fix B and alter f_c, which corresponds the situation where t_p is fixed, we can draw some explicit conclusions on how FB influences a receiver's performance. That is, monocycles with smaller FB, have better SNR gain but worse sync-error and multipath resistance abilities.
- If f_c is fixed and B is varied, according to the relation between autocorrelation and power spectrum, we can conclude that signals with smaller FB have better sync-error resistance ability, which is opposite to that stated above.

TABLE II

FRACTIONAL BANDWIDTH FOR MONOCYCLES

	$t_p = 0.7531$			$t_p = 0.5$	
n	2	5	14	2	5
FB	166%	121%	77%	166%	121%

- If both B and f_c vary, things become more complex as observed with Gaussian monocycles with diverse t_p and fixed n.

So it is meaningless to discuss a receiver's performance relating to signal's FB directly. However, FB is critical to the design of RF, antenna and some other parts of a UWB transceiver. A system with small FB is easier to design and implement. For Gaussian monocycles, a consideration exists between interference resistance and SNR gain when FB is fixed, as monocycles with same n have equal FB. Small t_p means better time-resolution and high SNR gain in a mulitple-user channel, while larger t_p means high interference resistance ability.

V. Conclusions

We have demonstrated that pulses have notable impact on the performance of correlator receiver and the effect can be observed from the autocorrelation function, especially the mainlobe. The autocorrelation is highly related to the SNR gain of the output and interference resistance ability. Specific to Gaussian monocycles,

- pulses with larger n means higher SNR gain in single user and asynchronous multiple access channel but inferior interference resistance ability;
- pulses with smaller t_p means higher SNR gain in asynchronous multiple access channel but inferior interference resistance ability.

This is quite different to DS-CDMA where the performance primarily depends on the correlation of PN sequences. This difference primarily results from the signal's low duty cycle and PPM method. To improve the performance of UWB systems, we concentrated on the construction of pulses and TH sequences with good correlation properties and lower and flat spectra, and now we have another factor to consider, that is the shape of a pulses' autocorrelation.

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