

# Phyz Examples: Rotation

## Smooth Operations: Kinematics

1. [Rot UM] How long does it take for a record spinning at 33.3 rpm to rotate through 100 radians?

$$1. \omega = 33.3 \text{ rev/min} \times 2 \pi \text{ rad/rev} \times 1 \text{ min}/60 \text{ s}$$

$$\omega = 3.49 \text{ rad/s} \quad \theta = 100 \text{ rad} \quad t = ?$$

$$\omega = \theta/t$$

$$t = \theta/\omega$$

$$t = 100 \text{ rad} / 3.49 \text{ rad/s}$$

$$t = \underline{28.7 \text{ s}}$$

3. [Rot UAM] A tire on my PhyzVan had an angular acceleration of  $-5 \text{ rad/s}^2$ . If the wheel was originally turning at  $80 \text{ rad/s}$  and turned through  $500 \text{ rad}$  during the deceleration, what was the final speed of the wheel?

$$3. \theta = 500 \text{ rad} \quad \omega_0 = 80 \text{ rad/s} \quad \omega = ?$$

$$\alpha = -5 \text{ rad/s}^2 \quad t = \text{WC!}$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\omega = \sqrt{\omega_0^2 + 2\alpha\theta}$$

$$\omega = \sqrt{(80 \text{ rad/s})^2 + 2(-5 \text{ rad/s}^2)(500 \text{ rad})}$$

$$\omega = \underline{37 \text{ rad/s}}$$

2. [Rot UAM] What is the angular acceleration of a compact disc that begins at rest and accelerates to  $50 \text{ rad/s}$  in  $1.5 \text{ s}$ ?

$$2. \theta = \text{WC!} \quad \omega_0 = 0 \quad \omega = 50 \text{ rad/s} \quad \alpha = ? \quad t = 1.5 \text{ s}$$

$$\omega = \omega_0 + \alpha t$$

$$\alpha = \omega/t$$

$$\alpha = 50 \text{ rad/s} / 1.5 \text{ s}$$

$$\alpha = \underline{33.3 \text{ rad/s}^2}$$

4. [Rolling] An oil barrel ( $r = 40 \text{ cm}$ ) rolls on a level surface at  $6 \text{ m/s}$ . What is the barrel's angular speed?

$$4. v = 6 \text{ m/s} \quad r = 0.4 \text{ m} \quad \omega = ?$$

$$v = r\omega$$

$$\omega = v/r$$

$$\omega = 6 \text{ m/s} / 0.4 \text{ m}$$

$$\omega = \underline{15 \text{ rad/s}}$$

## Smooth Operations: Dynamics 1

5. How far must a force of  $50 \text{ N}$  be placed along a wrench so that a torque of  $10 \text{ N}\cdot\text{m}$  can be achieved?

$$5. F = 50 \text{ N} \quad \tau = 10 \text{ N}\cdot\text{m} \quad r = ?$$

$$\tau = r \times F$$

$$r = \tau/F$$

$$r = 10 \text{ N}\cdot\text{m} / 50 \text{ N}$$

$$r = \underline{0.2 \text{ m} = 20 \text{ cm}}$$

7. How much torque is needed to angularly accelerate a  $3 \text{ kg}\cdot\text{m}^2$  fan blade at  $12 \text{ rad/s}^2$ ?

$$7. I = 3 \text{ kg}\cdot\text{m}^2 \quad \alpha = 12 \text{ rad/s}^2$$

$$\tau = I\alpha$$

$$\tau = 3 \text{ kg}\cdot\text{m}^2 \cdot 12 \text{ rad/s}^2$$

$$\tau = \underline{36 \text{ N}\cdot\text{m}} \quad [\text{The N}\cdot\text{m here are NOT Joules!}]$$

6. What is the mass of a basketball whose diameter is  $30 \text{ cm}$  and whose moment of inertia is  $0.0075 \text{ kg}\cdot\text{m}^2$ ?

$$6. R = D/2 = 0.15 \text{ m} \quad I = 0.0075 \text{ kg}\cdot\text{m}^2 \quad M = ?$$

$$I = (2/3)MR^2 \quad [\text{hollow sphere}]$$

$$M = (3/2)I / R^2$$

$$M = (3/2) 0.0075 \text{ kg}\cdot\text{m}^2 / (0.15 \text{ m})^2$$

$$M = \underline{0.5 \text{ kg}}$$

8. What is the combined rotational inertia of Jenny and the rotating stool she's sitting on if a torque of  $20 \text{ N}\cdot\text{m}$  causes an angular acceleration of  $2 \text{ rad/s}^2$ ?

$$8. \tau = 20 \text{ N}\cdot\text{m} \quad \alpha = 2 \text{ rad/s}^2$$

$$\tau = I\alpha$$

$$I = \tau/\alpha$$

$$I = 20 \text{ N}\cdot\text{m} / 2 \text{ rad/s}^2$$

$$I = \underline{10 \text{ kg}\cdot\text{m}^2}$$

## Smooth Operations: Dynamics 2 & 3

9. What is the angular momentum of a 20 g, 11.8 cm compact disc spinning at 500 rpm?

$$9. m = 20 \text{ g} = 0.020 \text{ kg}$$

$$r = d/2 = 5.9 \text{ cm} = 0.059 \text{ m}$$

$$\omega = 500 \text{ rev/min} \times 2 \text{ rad/rev} \times 1 \text{ min}/60 \text{ s}$$

$$\omega = 52.4 \text{ rad/s}$$

$$L = I\omega = (1/2)mr^2 \cdot \omega$$

$$L = (1/2) 0.020 \text{ kg} \cdot (0.059 \text{ m})^2 \cdot 52.4 \text{ rad/s}$$

$$L = \underline{0.0018 \text{ kg}\cdot\text{m}^2/\text{s}}$$

11. To what angular speed did Jearl accelerate the merry-go-round if its rotational inertia was  $300 \text{ kg}\cdot\text{m}^2$ ?

$$11. KE = 754 \text{ J} \quad I = 300 \text{ kg}\cdot\text{m}^2$$

$$KE = (1/2)I\omega^2$$

$$\omega = \sqrt{2KE / I}$$

$$\omega = \sqrt{2 \cdot 754 \text{ N}\cdot\text{m} / 300 \text{ kg}\cdot\text{m}^2}$$

$$\omega = \underline{2.24 \text{ rad/s}}$$

10. How much work does Jearl do on the merry-go-round if he applies a torque of  $120 \text{ N}\cdot\text{m}$  while accelerating it through  $2\pi \text{ rad}$ ?

$$10. \tau = 120 \text{ N}\cdot\text{m} \quad \theta = 2 \text{ rad}$$

$$W = \tau\theta$$

$$W = 120 \text{ N}\cdot\text{m} \cdot 2 \text{ rad}$$

$$W = \underline{754 \text{ J}} \quad [\text{Yes, the radians disappear and the N}\cdot\text{m become Joules. Weird, but true!}]$$

12. What is the angular speed of a bit on a 500 W electric drill that provides  $6.25 \text{ N}\cdot\text{m}$  of torque?

$$12. P = 500 \text{ W} \quad \tau = 6.25 \text{ N}\cdot\text{m}$$

$$P = \tau\omega$$

$$\omega = P/\tau$$

$$\omega = 500 \text{ W} / 6.25 \text{ N}\cdot\text{m}$$

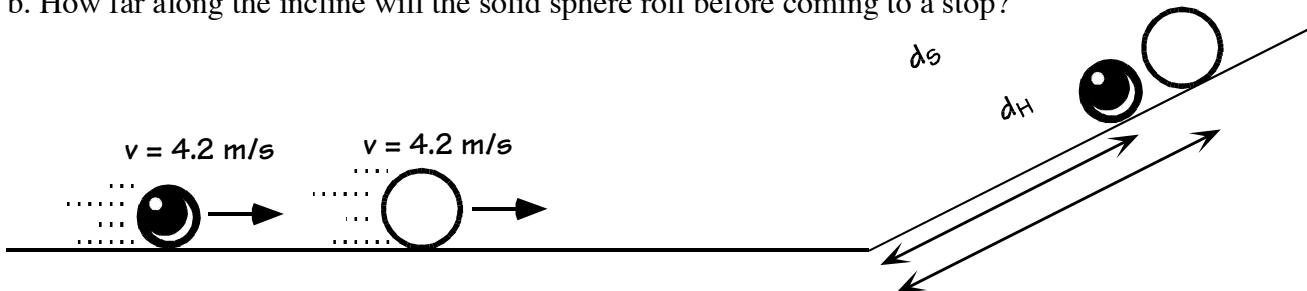
$$\omega = \underline{80 \text{ rad/s}}$$

## Welcome to the Real World Example

13. A hoop (hollow cylinder) and a solid sphere are rolling along a level surface at  $4.2 \text{ m/s}$  when they encounter an incline of  $27^\circ$ . Notice that I'm not saying what the mass or radius of either object is!

a. How far along the incline will the hoop roll before coming to a stop?

b. How far along the incline will the solid sphere roll before coming to a stop?



$$13a. v = 4.2 \text{ m/s} \quad \theta = 27^\circ \quad I = mr^2 \text{ (hoop)}$$

height above level surface

$$KE_{\text{bot}} = PE_{\text{top}}$$

$$KE_{\text{lin}} + KE_{\text{rot}} = PE_{\text{top}}$$

$$(1/2)mv^2 + (1/2)I\omega^2 = mgh$$

$$(1/2)mv^2 + (1/2)(mr^2)(v/r)^2 = mgh$$

$$(1/2)mv^2 + (1/2)(mv^2) = mgh$$

$$v^2 = gh$$

$$h = v^2/g$$

distance along incline

$$d_H = h/\sin\theta$$

$$d_H = v^2/g\sin\theta$$

$$d_H = (4.2 \text{ m/s})^2 / 9.8 \text{ m/s}^2 \sin 27^\circ$$

$$d_H = \underline{3.97 \text{ m}}$$

$$13b. v = 4.2 \text{ m/s} \quad \theta = 27^\circ \quad I = (2/5)mr^2 \text{ (SS)}$$

height above level surface

$$KE_{\text{bot}} = PE_{\text{top}}$$

$$KE_{\text{lin}} + KE_{\text{rot}} = PE_{\text{top}}$$

$$(1/2)mv^2 + (1/2)I\omega^2 = mgh$$

$$(1/2)mv^2 + (1/2)(2/5)(mr^2)(v/r)^2 = mgh$$

$$(1/2)mv^2 + (2/10)(mv^2) = mgh$$

$$(7/10)v^2 = gh$$

$$h = 7v^2/10g$$

distance along incline

$$d_S = h/\sin\theta$$

$$d_S = 7v^2/10g\sin\theta$$

$$d_S = 7(4.2 \text{ m/s})^2 / 10 \cdot 9.8 \text{ m/s}^2 \sin 27^\circ$$

$$d_S = \underline{2.78 \text{ m}}$$

Did you expect the same answer as for the hoop?