PhyzExamples: Advanced UCM & Gravity

Physical Quantities • Symbols • Units • Brief Definitions

Radius • R or r • m • Distance from the center of the circle to the center of the object in circular motion. If motion is circular, radius is constant.
Period • T • s • Time for one complete orbit. The reciprocal of frequency.
Frequency • f • 1/s or Hz (hertz) • Number of orbits per unit of time. The reciprocal of period.
Tangential Velocity • **v** / **Tangential Speed** • v • m/s • Velocity or speed of an object in circular motion. In UCM, tangential *speed* is constant while tangential *velocity* is always changing.
Centripetal Acceleration • a_C • m/s^2 • Acceleration of an object in circular motion directed radially inward (toward the center of the circle).
Centripetal Force • F_C • N • Force on an object in circular motion directed radially inward (toward the center of the circle).
Gravitational Force • F_G or W • N • Attractive force between two objects with mass (due only to their mass).
Universal Gravitation Constant • G • Nm^2/kg^2 • 6.67×10^{-11} Nm^2/kg^2
Astronomical Unit • AU • 93,000,000mi • The distance from the Earth to the Sun; this unit is used in measuring distances within the solar system.

Equations

$f = 1/T$ • frequency is 1 / period; $T = 1/f$ • period is 1 / frequency
$v = 2\pi R/T$ • tangential speed = two · pi · radius / period
$a_C = v^2/R$ • centripetal acceleration = tangential speed squared / radius
$F_C = mv^2/R$ • centripetal force = mass · tangential speed squared / radius
$R^3/T^2 = K$ • (Kepler's Law of Harmony) • radius cubed / period squared = Kepler's constant (Kepler's constant is different for different orbital systems)
$F_G = GMm/R^2$ • (Newton's Universal Gravitation) • gravitational force = universal gravitation constant · mass of one object · mass of the other object / radius squared

Smooth Operations Examples

1. What is the speed of a planet that orbits a star at 1 Gm once every 5 Ms?

1. $R = 1 \times 10^9$ m $T = 5 \times 10^6$ s $v = ?$
$v = 2\ R / T$
$v = 2\ (1 \times 10^9 \text{ m})/(5 \times 10^6 \text{ s})$
<u>$v = 1260$ m/s</u>

2. What is the speed of an object moving in a circle having a 2 m radius and experiencing a 12 m/s^2 centripetal acceleration?

2. $R = 2$ m $a_C = 12$ m/s^2 $v = ?$
$a_C = v^2/R$
$v = \ (a_C R) = \ (12 \text{ m/s}^2 \cdot 2 \text{ m})$ <u>$v = 4.9$ m/s</u>

3. What is the mass of an object moving at 4 m/s in a circle having a 3 m radius and experiencing a 24 N centripetal force?

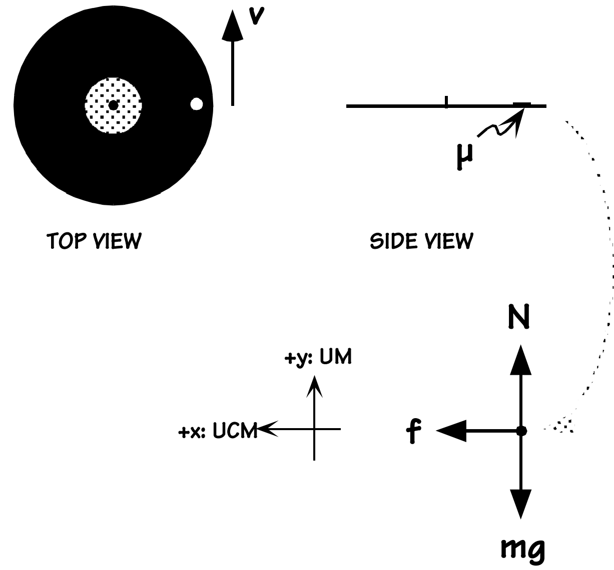
3. $v = 4$ m/s $R = 3$ m $F_C = 24$ N $m = ?$
$F_C = mv^2/R$
$m = F_C R/v^2$
$m = 24 \text{ N} \cdot 3 \text{ m} / (4 \text{ m/s})^2$ <u>$= 4.5$ kg</u>

4. How far is 80 kg Felix from 60 kg Bertha if they experience an attractive gravitation force of 100 nN?

4. $M = 80$ kg $m = 60$ kg $F_G = 100 \times 10^{-9}$ N $R = ?$
$F_G = GMm/R^2$
$R = \ (GMm/F_G)$
$R = \ (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \cdot 80 \text{ kg} \cdot 60 \text{ kg} / 100 \times 10^{-9} \text{ N})$
<u>$R = 1.78$ m</u>

Welcome to the Real World Examples

5. A nickel sits at the edge on an old style phonograph record (r = 15 cm). If the coefficient of static friction between the nickel and the record is 0.2, how fast can the nickel move before it flies off?



5. $F_x = ma_x$ $\quad$ $F_y = ma_y$

$f = mv^2/r$ $\quad$ $N - W = 0$

$\mu N = mv^2/r$ $\quad$ $N = mg$

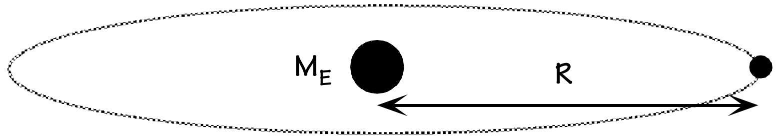
$\mu mg = mv^2/r$

$v = \ (\mu gr)$

$v = \ (0.2 \cdot 9.8 m/s^2 \cdot 0.15 m)$

$\underline{v = 0.54 m/s}$

6. How does the centripetal acceleration of the moon in its orbit compare to the gravitational acceleration imposed on the moon by the earth? The moon orbits at 3.82×10^8 m once every 27.3 d; the mass of the earth is 5.98×10^{24} kg.



6. Centripetal Acceleration of UCM:

$R = 3.82 x 10^8$ m $\quad$ $T = 27.3$ d $= 2.36 x 10^6$ s

$a_C = v^2/R$

$v = 2 \ R/T$

$a_C = 4 \ ^2 R/T^2$

$a_C = 4 \ ^2 \cdot 3.82 \times 10^8 \text{ m} / (2.36 \times 10^6 \text{ s})^2$

$\underline{a_C = 0.0027 \text{ m/s}^2}$

Gravitational Acceleration from Earth:

$R = 3.82 \times 10^8$ m $\quad$ $M_E = 5.98 \times 10^{24}$ kg

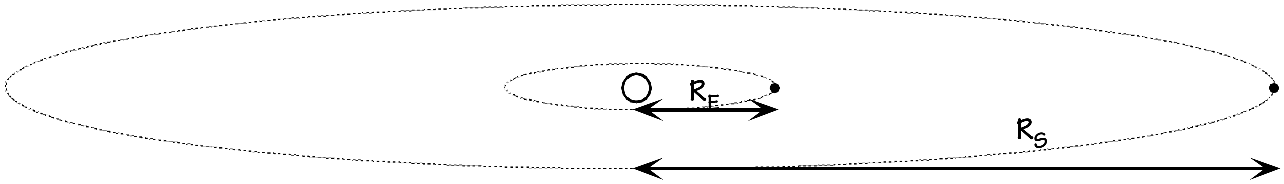
$g = GM_E/R^2$

$g = 6.67 \times 10^{-11} \text{ N·m}^2/\text{ kg}^2 (5.98 \times 10^{24} \text{ kg}) / (3.82 \times 10^8 \text{ m})^2$

$\underline{g = 0.0027 \text{ m/s}^2}$

With less information but more ingenuity, Isaac Newton determined that the equality of these two numbers confirmed his theory of gravity.

7. Suppose a ninth planet, Slowpoke, were found orbiting the Sun. Observations show that it has an orbital period of 412 years. How far is Slowpoke planet from the Sun?



7. Kepler's Law of Harmony (I choose to use AU's and years as my units.)

$R_E = 1 AU$ $\quad$ $T_E = 1 y$ $\quad$ $T_S = 412 y$ $\quad$ $R_S = ?$

$R_E^3/T_E^2 = R_S^3/T_S^2$

$R_S = {}^3 \ (R_E^3 T_S^2 / T_E^2)$

$R_S = {}^3 \ ((1 AU)^3 (412 y)^2 / (1 y)^2)$

$\underline{R_S = 55 \text{ AU} = 8.3 \times 10^{12} \text{ m}}$ (Compare to Earth at 0.15×10^{12} m.)

