

# PhyzGuide: Projectiles

## *motion in 2 dimensions*

Now that we are masters of one-dimensional kinematics, it is time to move into the world of two dimensions. With 2-D kinematics, we can study **projectile motion**.

Whereas in 1-D we could only study particles confined to moving in a straight line, we can now study objects that move along in an arc: baseballs, snowballs, tomatoes, eggs, spitwads, etc.

At first this may seem like an intimidating task: How can one keep track of such a complicated motion? The answer is a good news/bad news combination. The good news is that 2-D kinematics is incredibly simple: you don't need to learn *any* new equations. The bad news is that you may refuse to accept the good news above and insist on making 2-D kinematics more difficult than it is.

Here's the deal: Suppose a marble rolls horizontally along a table. It will continue to roll along at a constant speed (neglecting friction, as always). We already know how to describe *that* motion. It's a simple 1-D kinematics problem. The distance traveled by the marble is  $x = vt$ .

Suppose you were to drop the marble from the edge of the table. It would, of course, accelerate due to gravity. We already know how to deal with *this* 1-D motion as well. The distance traveled by the marble is  $y = \frac{1}{2}at^2$ .

Now suppose you roll a marble so that it rolls at a constant horizontal velocity and then rolls off the edge of the table. It becomes a projectile. It undergoes motions in two directions at once.

In the  $x$ -direction, the marble continues at its original  $v_x$ . Remember: there is nothing in the universe acting to accelerate the marble in the  $x$ -direction. The equation for motion in the  $x$ -direction remains  $x = v_x t$ .

In the  $y$ -direction, the marble is accelerating as if it had been dropped from rest. The equation for motion in the  $y$ -direction is  $y = \frac{1}{2}at^2$ .

A 2-D kinematics puzzle, then, is nothing more than two 1-D kinematics puzzles happening at the same time.

The trick to solving such puzzles is to think of the projectile as a "flying machine" with two pilots: an  $x$ -pilot, and a  $y$ -pilot.

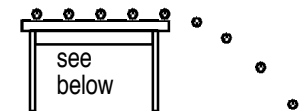
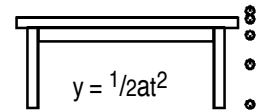
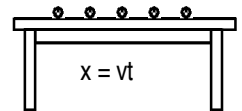
The  $x$ -pilot is programmed only for motion in accordance with  $x = v_x t$ .

The  $y$ -pilot is programmed for motion following the general equation  $y = v_{y0}t + \frac{1}{2}at^2$ .

Suppose you were asked, "How much time will pass between the point when the marble leaves the edge of the table and when it hits the floor?"

$y$ -pilot:  
(U.A.M.)  
 $y = \frac{1}{2}at^2$

To solve this, ask yourself, "Which pilot will be the first to know when the motion stops?" In this case, it is the  $y$ -pilot, so you can determine  $t$  from the  $y$ -pilot's equation. You can *not* determine  $t$  from the  $x$ -pilot's equation.



$x$ -pilot: (U.M.)  $x = v_x t$

