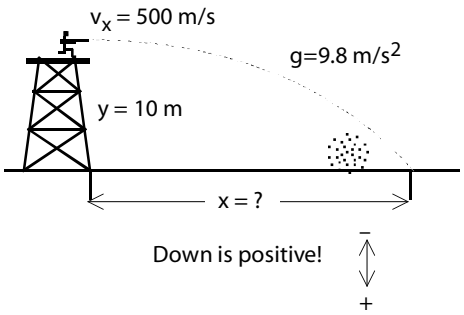
PhyzExample: Projectile Motion

A rifle is fired horizontally from a platform 10 m above level ground. The muzzle velocity (the speed at which the bullet emerges from the barrel of the gun) is 500 m/s. How far downrange will the bullet hit the ground?



Before doing anything else, we must decide "which way is up?" meaning: is the direction up positive or negative? Since our bullet is going DOWN and acceleration due to gravity is DOWN, I'm going to choose DOWN as positive. I could have chosen up as positive, but then y = –10 m and a = – 9.8 m/s^2, and I prefer positive numbers to negative numbers.

To solve the problem, first write down what you are GIVEN and what you can DEDUCE on a complete table of quantities.

x: UM	y: UAM
x = ?	y = 10 m
v_x = 500 m/s	v_{yO} = 0 m/s
	v_y = ?
	a = 9.8 m/s^2
t = ?	

THE FINE PRINT:

x: x is the unknown we're ultimately looking for.

v_x: Velocity in the x-direction does not change since there is NO natural acceleration in the x-direction.

t: Time for the flight is not associated with x or y—it is the same for both; and we don't know it.

y: y is +10 m because we chose down to be positive.

v_{yo}: Initial velocity in the y-direction is 0 m/s because the rifle is aimed HORIZONTALLY—no upward or downward component.

v_y: We don't know how fast the bullet will be travelling in the y-direction when it hits the ground, so v_y is unknown.

a: Acceleration in the y-direction is +9.8 m/s^2 because we chose down to be positive.

The question asks us how far downrange the bullet will land. This is a question about x. Motion in the x-direction is uniform motion (no acceleration), and is therefore described by:

$$x = v_x t.$$

This is the equation that will solve our problem. Unfortunately, we cannot use it yet. For although we ARE solving for x, and we DO know v_x, we do NOT know t—the time for the flight.

Question: which pilot will know first when the bullet hits the ground: the x-pilot or the y-pilot?
Answer: the y-pilot. It is the motion in the y-direction that limits the flight. The bullet is limited to travel only 10 m in the y-direction, whereas it could technically travel any distance in the x-direction.

So we can determine the time of flight by calculating how long it would take the bullet to fall 10 m.
THE TIME IT TAKES THE BULLET TO FALL 10 m IS THE TIME THE BULLET REMAINS IN THE AIR.
The general equation governing motion in the y-direction is:

$$y = v_{yO}t + \tfrac{1}{2}at^2$$

Since v_{yO} = 0 m/s, the equation can be simplified to

$$y = \tfrac{1}{2}at^2$$

To solve the x equation above, we need to know t. Solving the y equation for t gives · · · · ·

$$t = \sqrt{(2y/a)}$$

ALGEBRA:

$y = \tfrac{1}{2}at^2$

$2y = at^2$

$2y/a = t^2$

$t = \sqrt{(2y/a)}$

Since we were asked for x, not t, we DON'T plug in any numbers yet! Instead, we substitute the t we just found into the x equation, and only THEN plug in numbers.

$$x = v_x t$$
$$x = v_x \sqrt{(2y/a)}$$
$$x = 500 \text{ m/s} \quad \sqrt{((2)(10 \text{ m})/(9.8 \text{ m/s}^2))}$$
$$\boxed{x = 714 \text{ m}}$$