Phyz Job: Be Kepler for a Day!

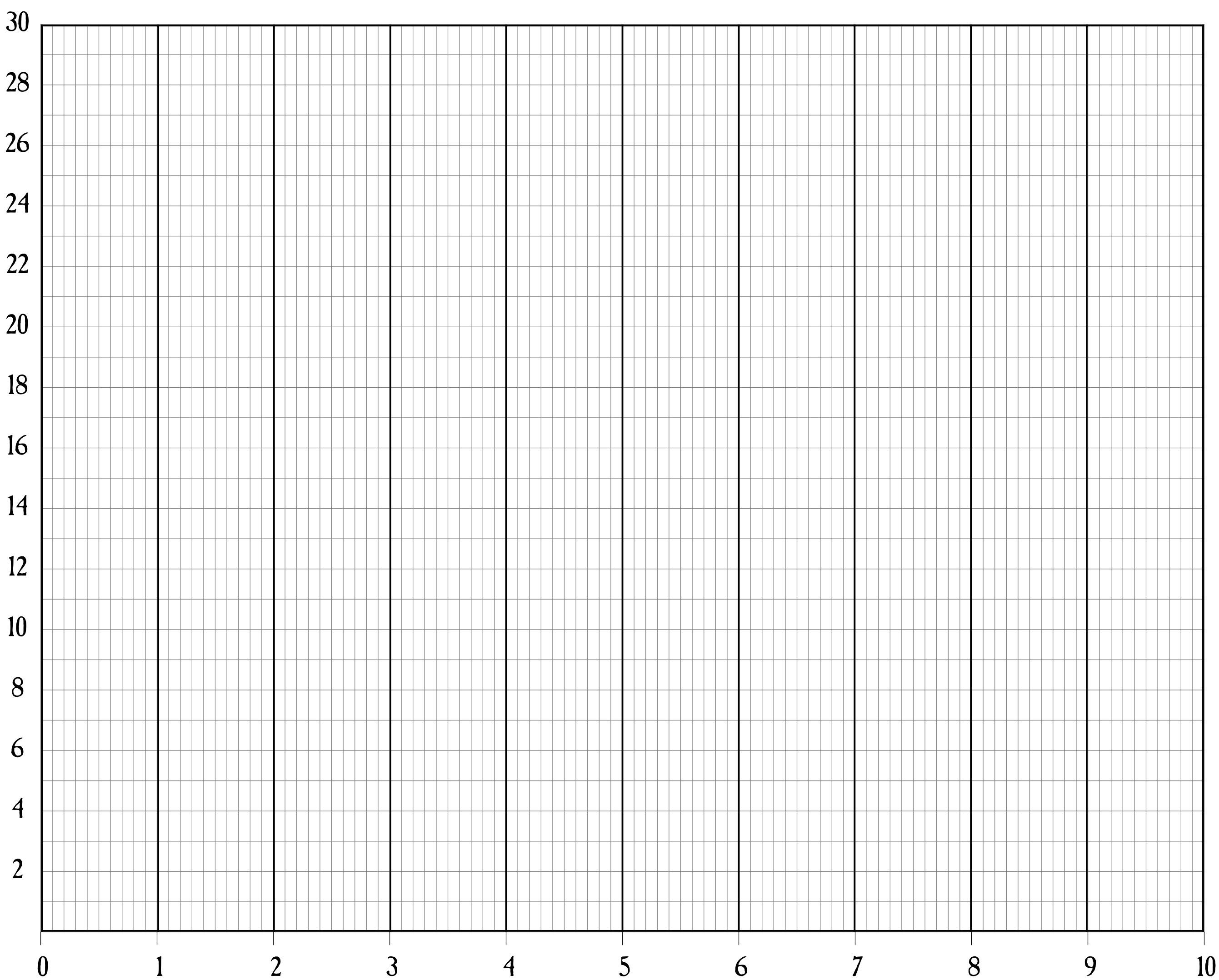
Part 1: Let the Graph be Made

name per date

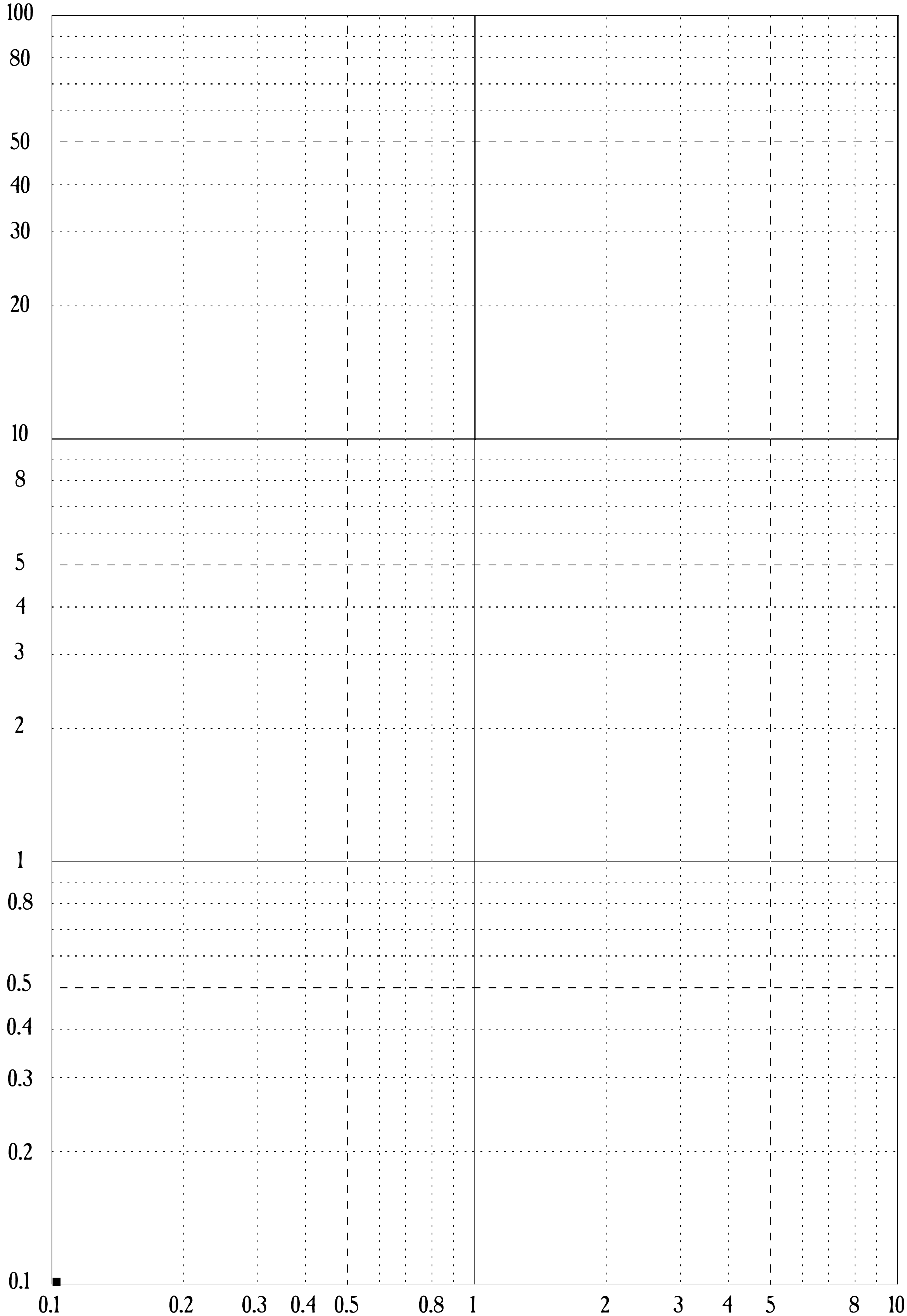
When Tycho Brahe died in 1601, Johannes Kepler obtained the Dane's highly accurate planetary data. After years of research and calculations, Kepler determined that all planets travel in ellipses (not circles, as had been thought since the the time of Aristotle). The sun was one focus of the elliptical orbit traced out by each planet. He also determined that planets sweep out equal areas in equal times, moving faster when nearer to the sun and slower when farther. Kepler was determined to find a relationship between the orbital radius of a planet and its orbital period. From Tycho's data, Kepler knew the numbers for the known planets.

Planet:	Mercury	Venus	Earth	Mars	Jupiter	Saturn
Radius	0.39AU	0.72AU	1.00AU	1.52AU	5.19AU	9.53AU
Period	0.241yr	0.615yr	1.00yr	1.88yr	11.9yr	29.5yr

To find a relationship, he graphed Period vs. Radius. Repeat his work and interpret the graph.



Kepler toiled for years with Tycho's data, searching for a connection between Radius and Period. When he learned of logarithms from Rene Descartes, he tried plotting the planetary data on log-log paper. Repeat his work and interpret the graph.



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Part 2: What Does It Mean?

name ______________________________ per ____ date ____

Had the graph of Period vs. Radius formed a straight line, we could apply the equation of a linear relationship, $y=mx+b$, to our case: $T=mR+b$. Had this been the case, the slope m would have been the constant of proportionality relating orbital period to orbital radius. The intercept value b would be the orbital period of a planet whose orbital radius is zero.

However, the plot of Period vs. Radius does not yield a straight line. The relationship between T and R is not linear.

The plot of Period vs. Radius on a log-log graph does yield a straight line. Similarly, a plot of $\log T$ vs. $\log R$ on regular graph paper would yield a straight line. What does it mean?

It means the relationship between period and radius is a *power function*. The general form of a power function is $y=Cx^n$, where C is a coefficient and n is the exponent (power) to which x is rasied. Logarithms allow us to winnow this equation down to something we can analyze with our understanding of the linear function. Here's how.

$y = Cx^n$	
$\log y = \log C + n\log x$	Take the logarithm of this equation
$y^* = C^* + nx^*$	Replace "log" with * notation to reduce clutter. So $\log y$ becomes y^*, etc.
$y^* = nx^* + C^*$	Rearrange
	This is a linear relationship of the form $y=mx+b$.

The slope of a plot of such a relation would give the exponent to which x is raised in its relation to y.

In our case, such a process gives the relation $T^* = nR^* + C^*$, where n is the exponent to which radius R is raised in its relation to orbital period T.

Determine the slope of the line plotted on the log-log graph paper. Use a ruler and find the slope by forming a triangle with the line as the hypotenuse and measuring the rise and run with the ruler. DO NOT USE NUMBERS/MEASURES FROM THE LOG-LOG AXES.

1. What is the slope of the line on the log-log graph?

2. So the equation of the line on the log-log graph is $T^* =$ _____ $R^* + C^*$. The value of the constant C is not important to us at this moment, although it is not difficult to find. With this data, the value of C is 1.

3. Translating our * notation back into log notation, we have $\log T =$ _____ $\log R + \log 1$.

4. Taking the antilog of the equation, we get $T = 1 \times R$. Dropping the 1 from our notation, we get $T = R$.

5. Square both sides of the equation. $T^2 = R$.

This is Kepler's Third Law, the Law of Harmony. If the units of measure had been meters and seconds instead of AU's and years, the relationship would be different only in the coefficient. The coefficient would have had a value other than one. So in general, the Law of Harmony is written $R^3/T^2 = K$, where K is a constant.

The Law of Harmony holds true for any system of satellites. The moons of Jupiter, the particles that form Saturn's rings, the network of satellites orbiting the earth; all satellites in a given system have a common ratio of R^3/T^2. The specific value of the ratio is *different* for each system. Isaac Newton pondered the reason for this, and drew a powerful conclusion. As we shall see. Stay tuned...