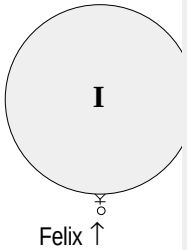
PhyzJob: From the Centre of the Earth to the Surface… *and Beyond!*

I. Suppose Felix Fizix (m=60kg) was standing on the surface of the earth as shown in the diagram.



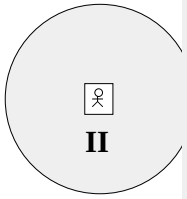
1. If Felix dropped a coin, which way would it fall and with how much acceleration? Explain.

UP! Toward the center of the earth.

2. If Felix stepped on a bathroom scale, what would the reading on the scale (Felix's weight) be? Explain.

588N (W=mg)

II. Suppose Felix Fizix was standing in a chamber at the center of the earth as shown in the diagram.



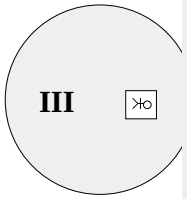
1. If Felix dropped a coin, which way would it fall and with how much acceleration? Explain.

Wouldn't fall: would float!

2. If Felix stepped on a bathroom scale, what would the reading on the scale (Felix's weight) be? Explain.

0: he's weightless!

III. Suppose Felix Fizix was standing in a chamber halfway between the center and the surface of the earth as shown in the diagram.



1. If Felix dropped a coin, which way would it fall and with how much acceleration? Explain.

Toward the center of the earth.

2. If Felix stepped on a bathroom scale, what would the reading on the scale (Felix's weight) be? Explain.

I dunno. Maybe half his weight at the surface?

Do not read this until you have answered the questions above!

Remember that every bit of matter in Felix is attracted to every bit of matter in the earth.

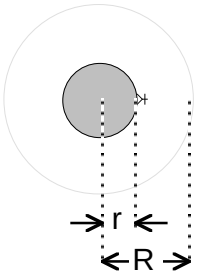
In situation I, Felix is pulled only toward the center of the earth.

In situation II, Felix is pulled in all directions by all the bits of matter in the earth. The resulting effect is (recall the "Leggo my Eggo" principle) that the net force is zero: Felix is weightless!

In situation III, there are more bits of matter pulling Felix toward the center of the earth than toward the surface. Felix has weight, but not as much as when he is on the surface of the earth. So how much *would* Felix weigh here?

Newton considered this problem mathematically and found a solution. (It is said that he invented calculus to solve this problem. Relax, we'll deal with the non-calculus version.)

Newton determined that the force acting on Felix in situation III would be the same as if he were standing on a planet consisting only of the mass "beneath" him as shown on the diagram.



The whole earth has a mass corresponding to its full radius *R*, when Felix is at a distance *r* from the center of the earth, his weight is as if he were on a planet with the same density as the earth, but with a radius *r* instead of *R*.

So how much does an object weigh when it is somewhere between the center of the earth and the surface of the earth?

1. What is the relation between mass, density, and volume? $D = M/V$

2. What is the volume *V* of a sphere of radius *r*? $V = 4/3\ \pi R^3$

3. What is the mass *M* of that sphere if its density is *D*? $M = 4\pi R^3 D/3$

4. What is the equation for the law of universal gravitation? $F = GMm/R^2$

5. Substituting the expression for *M* in Q3 into the equation in Q4 gives: *(Be sure to simplify!)* $F = 4\pi GDRm/3$

6. Determine the weight of a 1kg object at $r = R_E/2$, R_E , $2R_E$, and $3R_E$ and plot the points on the graph below. Based on the equation in Q5, sketch the line from $r = 0$ to $r = R_E$. Using the expression in Q4, sketch the line from $r = R_E$ to $r = 3R_E$.