Just what is vagueness?

OTÁVIO BUENO & MARK COLYVAN

1. Introduction

The standard definition of vagueness is in terms of borderline cases. But what is a borderline case? The obvious answer is that a borderline case is one which neither falls under the predicate in question nor does not fall under the predicate. One problem with this definition is that it presupposes that borderline cases involve gaps and not gluts. A definition of vagueness should not prejudice the question of how best to deal with it, and yet the standard definition rules out a paraconsistent approach to vagueness right from the start (such an approach insists that vagueness involves gluts not gaps).

In this paper, we explore some of the ways in which vagueness might be spelled out and consider problems for each of them. In the end, we propose a non-question begging definition of vagueness and discuss some of the advantages and shortcomings of this alternative.

2. Two approaches to the sorites

Most philosophers are rather sympathetic to approaches to the sorites paradox that involve truth-value gaps. But hardly anyone is sympathetic to paraconsistent (or glutty) approaches to vagueness. We find this rather surprising, particularly in light of the fact that the most popular gappy approach—supervaluationalism—is formally equivalent to a glutty subvaluational approach (see, e.g., Hyde 1997).

There may, of course, be many (non-formal) reasons for philosophers' preference for gaps over gluts, at least with respect to vagueness. We suggest that one of the reasons is that the question has been begged against truth-value gluts in favour of truth-value gaps in the definition of 'vagueness' and 'borderline case'.

For ease of presentation, it will be convenient to focus on a particular instance of the sorites paradox:

(P1) A one day old is a non-adult. (Base case.)

(P2) If an n-day old is a non-adult, then a n+1-day old is a non-adult. (Inductive clause.)

Therefore (by mathematical induction):

A 21915-day (or 60 year) old is a non-adult.

Undoubtedly, the front runner amongst philosophers and logicians, at least, for a satisfactory solution to sorites paradoxes such as the one above is the supervaluational account (see, e.g., Keefe 2000). According to supervaluationists, many claims about borderline cases are neither true nor false. So the sentence 'A 5844-day (or 16 year) old is a non-adult' is taken to be neither true nor false.[1] The supervaluationist, thus, rejects the inductive clause (P2) of the sorites argument. Moreover, she does this without accepting that there is a sharp cut-off between adulthood and non-adulthood. The virtues of supervaluationalism are well known. One of these alleged virtues is that supervaluationalism preserves classical logic, in the sense that all the theorems of classical logic are theorems of supervaluational logic. In particular, although bivalence is given up by the supervaluationalist, excluded middle is still a theorem. So much for supervaluationalism. Now to a much underappreciated alternative: subvaluationalism.

According to subvaluationalism, claims about borderline cases are both true and false (see Hyde 1997). So the sentence 'A 5844-day (or 16 year) old is a non-adult' is taken to be both true and false. The supervaluationist, thus, also rejects the inductive clause (P2) of the sorites argument. And again she does this without accepting that there is a sharp cut-off between adulthood and non-adulthood. The virtues of this approach are less well known, so let us mention a few of them. First, this approach also preserves all the theorems of classical logic (if this is a virtue) in much the same way that supervaluationalism does. In particular, excluded middle and the law of non-contradiction are both theorems.[2]

Indeed, the virtues of the subvaluational approach are much the same as those of the supervaluational approach. This is no accident, since subvaluationalism and supervaluationalism are formally duals of one another (Hyde 1997). Indeed, given this formal symmetry, there is very little to choose between them. Certainly there can be no formal grounds for choosing one over the other.[3]

But why, then, is the supervaluational approach considered by almost everyone as highly plausible and the subvaluational approach as utterly implausible? There are, we think, many reasons for why philosophers prefer gaps over gluts generally and, in particular, in the case of vagueness. None of these reasons, however, in our view is a good reason. One significant reason for most philosophers preferring a gappy, supervaluational approach to vagueness over a glutty, subvaluational approach is that the question of how best to deal with vagueness is effectively begged against the subvaluational approach in the very definition of vagueness. We will explore this issue in the remainder of this paper.

3. What is a borderline case? The usual account

Vagueness is usually defined in terms of borderline cases. The usual definition of a vague predicate is:

---

[1] Many claims about borderline cases are true. For example, the sentence 'a 5844-day old is a non-adult or it is not' is true.

[2] One can also argue that, although the subvaluational semantics are non-classical, they are bivalent, in the sense that there are only two truth values—the True and the False. What is given up is the classical constraint that every proposition takes one and only one of these two truth values. We won't push this point though because (i) we think it is more natural to think of the case where a proposition is both true and false as a case where the proposition in question is taking a third truth value, (ii) it can also be said that supervaluationalism is bivalent, in the sense that here too there are only two truth values, but some propositions do not take either of these values.

[3] Though there might be other more philosophical grounds. See Beall and Colyvan (2001) and Hyde (2001) for more on this.

Definition 1. A predicate is vague iff it permits borderline cases.

But this doesn't help us get a grip on vagueness until we understand what a borderline case is. And for this there are a few definitions on offer. Most of these, however, presuppose some kind of gappiness. For example, it is often claimed that a borderline case is one to which the predicate in question neither applies nor does its complement. Notice that on this account, if *a* is a borderline case of adult, say, then '*a* is an adult' is (plausibly) neither true nor false. And it is not the case that '*a* is an adult' is both true and false. In short, this definition presupposes that vagueness is an essentially gappy phenomenon. Glutty approaches are ruled out by fiat.

Or consider the definition of borderline case as one where the predicate in question neither determinately applies nor determinately does not. Again we see that this introduces gaps—this time in the application of the 'determinately applies' predicate, rather than the vague predicate itself. But again we rule out by fiat the approach of considering a borderline case of a vague predicate as a case where the predicate in question determinately applies and determinately does not apply.

Another account of borderline case defines them in terms of there being no fact of the matter regarding the application of the predicate in question (Sainsbury 1995). It is interesting to note that this type of definition has been objected to by philosophers such as Williamson (1994) and Sorensen (1988) who are sympathetic to the epistemic approach to vagueness. According to the latter account, there *is* a fact of the matter about the application of vague predicates to borderline cases; it's just that we don't know whether they apply or not. Moreover, we can never know whether they apply or not. Now we're no fans of this approach, but we agree that it should not be ruled out by the way the problem is set up. So this definition is inadequate for the same sorts of reasons that definitions that imply gaps are unacceptable: they rule out a contender by fiat. Another problem with this definition is that it does not distinguish between a borderline case and a partially defined predicate. Consider the predicate 'is a detective'. Holmes satisfies this, Moriarty does not and others in the Doyle stories neither satisfy nor do not satisfy. But this has nothing to do with vagueness—it's simply that the domain in question is incomplete. What is it, on this account, that distinguishes vague predicates from other incomplete predicates?

We won't go through all the definitions of 'borderline case', but one more is worth considering. It is often said that a borderline case is one where it is not clear whether or not the predicate under consideration applies (Keefe and Smith 1997). This definition seems straightforwardly inadequate simply because it does not rule out epistemic uncertainty for non-vague predicates. For example, we're uncertain whether the predicate 'is taller than 170 cm' applies to Pelé, simply because we don't know his exact height, but this doesn't mean that the predicate 'is taller than 170 cm' is vague—clearly it is not. But, in this case, someone knows whether Pelé is taller than 170 cm. However, there are many other cases like this where no one knows and yet the predicate in question is still not vague.[4]

4. Another dead end: failure of mathematical induction

[4] Consider the predicate 'has more white blood cells than Pelé at midday US EST on May 8, 2004'. This predicate, we take it, is not vague and yet unless Pelé was having a blood test done at that exact moment, no one will know his white cell count, and so it will not be clear to anyone when to apply the predicate in question.

What we're after is a definition of 'vagueness' that does not beg any questions about how vagueness is best treated. In particular, we need an account of vagueness that does not presuppose that it is essentially a gappy phenomenon. As motivation for an alternative account, consider once again the argument at the beginning of section 2. Note that it is an argument employing mathematical induction that leads from (apparently) true premises to a false conclusion due to the vagueness of the predicate in question. The failure of mathematical induction, thus, seems like a promising way to characterise vagueness:

Definition 2. A predicate is vague iff it is a predicate for which mathematical induction fails.

Of course, we need to spell out what is meant by 'fails' here. There are two alternatives: (i) the particular mathematical induction argument in question is invalid, or (ii) the argument is unsound (due to a false premise). But either way we end up begging the question against some account or other.[5]

Let's suppose we take option (i). This begs the question against many approaches to the sorites (e.g., Williamson's epistemicism, and supervaluationalism, both of which reject one of the premises of the inductive soritical argument). It won't help to claim that we meant (ii) rather than (i) either. Since that would be begging the question against fuzzy approaches, which reject the reasoning employed in sorites arguments—i.e., they reject the validity of mathematical induction when applied to vague predicates.

But what of a disjunctive understanding of 'fails' in definition 2, so that induction fails in the sense that *either* mathematical induction is invalid *or* it's unsound (or perhaps both)? So the epistemicist, as well as the supervaluation and the subvaluation theorists will all agree that a given predicate is vague because the relevant case of mathematical induction is unsound, whereas the fuzzy theorist will say that the predicate is vague because the case of mathematical induction in question is actually invalid. So far so good. The problem is that there are those, such as Peter Unger (1979), who accepts the truth of the premises, the validity of the reasoning, and hence the paradoxical conclusion. We've just begged the question against *this* account! Perhaps we can't please everyone here. But unless a non-question begging definition of vagueness is forthcoming, we will not be in a position to identify the nature of the phenomenon we are trying to accommodate.

However, there is another, more serious problem with definition 2: as a definition of vagueness it both captures too much and it doesn't capture enough. The first difficulty is that if we define a predicate to be vague just in case it is one for which mathematical induction is either invalid or unsound, this does not uniquely pick out vague predicates. Consider the argument:

(P1) 2 is a prime number.

(P2) If n is prime, n+1 is also prime.

[5] Thanks to Graham Priest for raising this point.

Therefore:

All natural numbers greater than or equal to 2 are prime.

This is clearly an example of a failed mathematical induction (failed in the sense that the conclusion is clearly false). Moreover, it fails because one of the premises is false (namely, P2), but this would suggest that according to the above definition of 'vagueness', the predicate 'is a prime number' is vague. But this is clearly wrong. 'Is a prime number' is a paradigmatic sharp predicate. In short, definition 2 is not restrictive enough.

Moreover, definition 2 does not apply to non-numerical vagueness. Examples of non-numerically vague predicates are 'is a religion' and 'is a game'. Such predicates do not have a natural numerical ordering from cases where they apply, through borderline cases, to cases where they do not apply. Yet they clearly admit of borderline cases (e.g., certain ritualistic activities, such as following Brazillian soccer, we take it qualify as borderline cases of religions). The problem here is that while non-numerically vague predicates admit borderline cases, they do not seem to support mathematical induction style sorites arguments. So, it would seem that definition 2 is of little use.[6]

5. Where from here? A positive proposal

So far we have no non-question begging definition of 'vagueness'. Still we all know what vagueness is, don't we? Why isn't it enough to say 'we recognise it when we see it'? What is to be gained by providing a definition? As Stewart Shapiro (forthcoming) notes, it is rather odd that philosophers debate the correct account of vagueness and yet no one can say what it is that the various contenders provide accounts of. Worse still, without a definition of 'vagueness' it is not even clear that the various theories are theories of *the same phenomena*. There's no doubt that a definition of vagueness is desirable. Equally clear, however, is that an inadequate or question-begging definition is undesirable. Indeed, we suggest that the definitions we've considered thus far in this paper are all inadequate and are worse than no definition at all.

We have a positive proposal, though. To motivate it, it is worth stepping back from the issue of a definition of vagueness and ask why vagueness matters. The importance of vagueness lies in the fact that it results in paradox. Bearing this in mind, our proposal is to suggest that instead of defining vagueness in terms of borderline case, the focus should be on sorites arguments. We suggest as a working definition: *a predicate is vague just in case it can be employed in a sorites argument*.[7] Of course, we now need an independent account of what a *sorites argument* is, but we might do this by simply stating that *it is an argument by degrees with premises that appear to be true, but with a conclusion that appears to be false*.[8] Then we simply list the various diagnoses of

[6] Interestingly, the usual definitions of 'borderline case' accommodate the non-numerically vague just as readily as the numerically vague. We will return to the issue of non-numerical vagueness below.

[7] This suggestion is already in the literature. For example, Stewart Shapiro (forthcoming) is content to use a definition of vagueness along these lines. But we've seen no systematic defence of this approach.

[8] Note that with this formulation of a sorites argument, we can rule out failed cases of mathematical induction as not being soritical, given that typically at least one of the premises of the argument doesn't appear to be true. This is the

what goes wrong (i.e., a false premise, the reasoning is invalid, or the conclusion is, despite appearances, true).

The advantages of this approach is that it doesn't beg any questions against any account of vagueness, since *all* parties in the debate agree that vague predicates are those that can be employed in a sorites argument. But this proposal seems to face a couple of problems. First, it is not clear that this account can accommodate non-numerical vagueness. Recall that non-numerical vagueness is the vagueness found in predicates like 'is a religion', where there are borderline cases but no natural numerical ordering between 'is a religion' and 'is not a religion'.[9] Because there is no natural ordering, it would seem that there can be no sorites argument, yet the predicate in question is vague nonetheless. Recall that we raised this problem for an earlier proposal: the proposal that vagueness is that which features in failed mathematical induction arguments. But there is an interesting difference between our proposal and this earlier one. A sorites argument is a particular type of argument by degrees, and the latter does not, in general, require all the resources required to run a mathematical induction argument.

In particular, mathematical induction requires a total ordering (typically provided by the natural numbers, but any totally-ordered set will do). The problem here is that with non-numerical vagueness we do not have a total ordering. To return to our example of religion, quite different activities may plausibly be thought to be equally religion like. Consider, for example, two borderlines cases of religion: following Brazilian soccer and following Italian soccer. Both activities have ritualistic behaviour, belief in entities (the star players) worthy of something bordering on worship, belief in extraordinary (if not supernatural) powers of these stars, and so on. It seems that neither of these is more or less religion like than the other. So, how do we construct an appropriate sorites series when all we have is a partial ordering?

First, we note that although a total ordering is required for mathematical induction arguments, and typical sorites arguments can be represented as mathematical induction arguments, the total ordering for sorites arguments is unnecessary—a partial ordering is sufficient. The sorites argument from religion to non-religion, say, must confine itself to activities that are totally ordered in a given path from religion to non-religion, and so typically there will be more than one such argument proceeding via a different path in the partially-ordered structure in question. But this non-uniqueness does not matter. All that matters is that there is *at least one* such sorites series available. So, to return to our example of religion, consider the following series of activities which (arguable) are totally ordered: Christianity, Buddhism, Brazillian soccer, Australian Rules Football, Minor League Baseball, Schoolyard play. Of course, before we could conduct a plausible sorites argument, we would need to fill in a few more activities between each of those above, but the idea, we take it, is clear enough. Unlike the usual sorites arguments from 'tall' to 'not tall', there are multiple routes from 'religion' to 'non-religion', but the fact that there is one

---

case, for instance, of the argument regarding prime numbers discussed in Section 4 above, in which (P2) is clearly false.

[9] What we are calling 'non-numerical vagueness' some refer to as multidimensional vagueness. The thought being that predicates like 'is a religion' are (numerically) vague along a number of dimensions, such as belief in a deity (or deities), worship of the deity (or deities), belief in supernatural powers (usually associated with the deity), ritualistic behavior, and so on. Even if each of these dimensions has a natural numerical ordering, there is no non-trivial way of reducing these multiple numerical scales to a *single* numerical scale. In technical parlance, multi-dimensional vagueness (or non-numerical vagueness) has only a partial ordering, not a total ordering, associated with it.

such route is enough. So, in the end, the account of vagueness we propose here can accommodate non-numerical vagueness, and we take this as a significant virtue of the proposal.

A final potential problem with our proposal for the definition of vagueness is that we need to be able to recognise sorites arguments when we see them. But this does not strike us as too serious, for surely we *are* able to recognise such arguments when we see them. Moreover, the account of sorites arguments suggested above also helps in recognising these arguments. To recognise a sorites, we should look for an argument by degrees whose premises appear to be true, but whose conclusion appears to be false. There's, of course, some vagueness involved here. For instance, what exactly counts as an *argument by degrees*? Rather than a difficulty, we take this to be an advantage of the proposal, since we are not ruling out higher-order vagueness by fiat (that is, we are not ruling out the vagueness of the term 'vague'). According to our proposal, those who deny higher-order vagueness ultimately will end up having to provide, say, a precise definition of 'argument by degrees'. Our proposal is not committed either to rule out or to endorse a priori the possibility that such a precise definition can be given, and in this sense, our account also captures why higher-order vagueness emerges. It's just the outcome of the way in which vagueness has been characterised. In addition, using the account above of a sorites argument, we can also identify cases of failed mathematical inductions as *not* being soritical, since as noted above, this failure typically involves at least one premise in the argument *not* appearing to be true.

Finally, our positive proposal regarding vagueness (basically, as a predicate that can be used in a sorites argument) is much better than the usual question-begging definition invoking borderline cases and taking these to be essentially gappy. Avoiding begging questions against even unpopular accounts of vagueness (like subvaluationalism) is highly desirable, and our proposal delivers that as well.

6. Conclusion

So, just what is vagueness? Our present proposal is that vagueness is whatever is expressed by vague predicates, and the latter are those that can be employed in sorites arguments. In turn, a sorites argument is an argument by degrees whose premises appear to be true and whose conclusion appears to be false. By not ruling out by fiat any account of vagueness, this proposal opens the way to an understanding of vagueness as a uniform phenomenon— even though, as the plurality of extant accounts of vagueness indicates, there's plenty of room for diversity, even within the framework advanced here.[10]

University of South Carolina
Columbia, SC 29208, USA
obueno@sc.edu

University of Queensland
Brisbane, Queensland 4072, Australia

[10] Our thanks go to the audience at the 2002 Australasian Association of Philosophy Conference, where an earlier version of this paper was presented. We are also indebted to Dominic Hyde and Graham Priest for very helpful conversations on the topic of this paper.

mcolyvan@uq.edu.au

References

Beall, JC and M. Colyvan, 2001. Heaps of Gluts and Hyde-ing the Sorites. *Mind* 110: 401–8.

Hyde, D. 1997. From Heaps and Gaps to Heaps of Gluts. *Mind* 106: 641–60.

Hyde, D. 2001. A Reply to Beall and Colyvan. *Mind* 110: 409–11.

Keefe, R. 2000. *Theories of Vagueness*. Cambridge: Cambridge University Press.

Keefe, R. and P. Smith 1997. *Introduction: Theories of Vagueness*. In R. Keefe and P. Smith, *Vagueness: A Reader*. Cambridge, Massachusetts: MIT Press, pp. 1–57.

Sainsbury, R.M. 1995. *Paradoxes*. 2nd edition. Cambridge: Cambridge University Press.

Shapiro, S. forthcoming. *Humpty Dumpty on Vagueness*. Oxford: Oxford University Press.

Sorensen, R.A 1988. *Blindspots*. Oxford: Clarendon Press.

Unger, P. 1979. There Are No Ordinary Things. *Synthese* 41: 117–154.

Williamson, T. 1994. *Vagueness*. London: Routledge.