Rational Expressions Unit Plan

Mr. Fahrenbacher

Math 80s

**Preface**

This unit plan was by far the most abstract of any unit I created during my student teaching experience. It was a difficult topic for me to motivate, as it is mostly setup for material my freshman students would not revisit until late their junior year. Interestingly enough, my students seemed to enjoy the topic, as it seemed that they were required to think less and follow more procedures. But to successfully deal with rational expressions, students need to do really think and understand what they are doing. So in the unit, there needed to be a balance between direct instruction on how to attack rational equations and how to think dynamically about these problems to deal with new and tricky situations. Each lesson in the unit focused first on what students already new and then expanded upon that knowledge through questioning to learn something new. This technique allowed for the mathematics to be built upon previous knowledge and make it seem more attainable to the students. At the same time, a fairly specific set of procedures for how to deal with rational equations was developed. Together, there is a good mix of discovery and extending knowledge as well as learning and practicing with new techniques and ideas, the best of both creative and rote learning.

**Plan**

Day One:

The first day of the unit focuses in on what students should already know: long division. Unfortunately, some students have not ever done, or don't remember how to do long division, so at least ten minutes should be allocated to reviewing the process, with students who remember guiding the teacher through what to do. Key things to link from long division to polynomial long division include: the importance of zeros, remainders, and the effects of

bringing down more than one number. Homework: 6 long division problems.

Day Two:

After the homework, many students should agree that polynomial long division is a pain. Drawing on that, they should be ready for synthetic division! The shortcut only works when dividing by a linear term, and simply showing a division problem with both techniques at the same time helps show why synthetic division works. An example or two of polynomial and synthetic division also can motivate how the synthetic solution should be interpreted. Homework: 2 long division and 4 synthetic division problems.

Day Three:

Now for something completely different. Instead of doing long or synthetic division, sometimes we get lucky and can avoid doing it by using cancelation. There is a lot of little things to review, including how to cancel like terms, why dividing by a fraction means to invert and multiply (how many 1/2 pies are there in 2 pies?), and how to factor out a negative. Homework: 1 long division, 2 synthetic, and 6 cancelation problems.

Day Four:

Day four is really really really easy to do. All that is covered is how to combine fractions with like terms. The easiest way to introduce the topic is by working with rational fractions with common denominators, as that is a very familiar topic. The one catch is subtracting fractions, which requires students to see that the negative needs to be distributed through a quantity. Homework: 2 cancelation problems and 6 combining fractions with like denominators problems.

Day Five:

Even the spaciest student could tell what is next - adding fractions with unlike denominators. Because mathematicians are lazy, we'd like to use what we

learned yesterday, but somehow we need to make the denominators common. Two things to show with rational fractions is that multiplying the top and bottom of a fraction by the same number doesn't change the fraction and that we want to use the least common denominator with our multiplications, not just any old denominator. Homework: 2 combining fractions with like denominators and 6 combining fractions with unlike denominators problems.

Day Six:

The complexity of the work increases with complex fractions, or fractions inside of fractions. Just like with fractions with uncommon denominators, we would like to use previous knowledge (invert and multiply) to solve this problem - let's find a common denominator of the bottom fraction and the top fraction! After an example using this technique, it's time to contrast it with a new technique: multiplying the top and bottom of the big fraction by the least common denominator of the little denominators. Low and behold, the process is faster! It's important to note that both techniques will work, and that sometimes students may find it easier to do one technique or the other in a given situation. Homework: 2 combining fractions with unlike denominators and 6 complex fractions problems.

Really good example: $\dfrac{1+\frac{1}{x}}{1-\frac{1}{x}}$.

Day Seven:

Lastly, before the quiz, there are partial fractions, an interesting process that is used quite a bit in calculus. We've gone from two fractions to one, wouldn't it be cool to go from one fraction to two (insert student groans)?! The process follows pretty naturally with a small amount of motivation. We want to break the fraction into two, so the natural thing to start with is what we know (two different denominators) and what we don't know (what are the two

numerators?). Following this, the next question is - what can go wrong with our equation (division by zero) and how might we fix it (multiply through by the denominators)? Then we need to remember our goal: we want to find the numerators (usually denoted as A and B). But we have one equation and three unknowns! How could we eliminate two of them, say x and B? The answer is to plug in something for x that makes the coefficient on B zero, and then solve for A, and the process is similar for finding B. After a couple examples, the process is pretty straight forward. Homework: 3 complex fractions and 3 partial fractions.

Day Eight:

Review day. Today we go over everything that will be on the quiz for the next day and things to watch out for. Most of the period is dedicated to examples on the board by students and the teacher.

Day Nine:

On the ninth day, there was a quiz covering everything from the previous eight days. To aid in the grading process, I created an exhaustive rubric that explains exactly where I assigned points.

**Reflections**

This unit seems like a fairly long, daunting task when written out on paper, but this perception is mostly because it is very well spread out. Breaking down the topics piece by piece was helpful for students who were having some trouble following the abstract nature of the topic. The oe thing I wish I had incorporated into the topic was more visuals of why we deal with rational equations the way we do, a not so trivial task. For one, the argument for why we invert and multiply is a golden opportunity to bring in food or other prop to use and demonstrate. In addition, I feel that many of the students were simply going with the flow, and even if they understood what we were doing, there was no real understanding of why would want to do this stuff. One possible constraint on this was that I didn't have a very strong sense of examples from pre calculus that I could use with my

students. In the next unit, we did begin to use these properties more in solving and graphing rational equations, so there was some justification in the end. I believe the quiz was very well constructed and I really think my students appreciated the extra time that went into creating a diagnostic rubric that pinpointed what problems they had when taking the quiz.

I am especially proud of the homework assignments, being motivated by Ken Indeck of Buffalo Grove High School. I had looked at the original plan for the unit, and the homework assignments were much longer, more than twice as long is some cases, and I didn't really see the point. If a student does thirty problems and gets them all wrong, I've just encouraged my student to do the mathematics the wrong way! If a student does thirty problems and does them all correctly, I don't see why they need to do thirty; ten or so would be fine. Observing the quiz scores after the unit, I was not disappointed - the scores did vary from high to low, but overall the class did very well even though the homework was significantly slashed. Limiting the homework also provided more time in class for discussion and more material, instead of wasting time going over the same type of question multiple times.

The use of group work during this unit during my execution and following the guidelines above was relagated to going over homework assignments near the end of the period for the most part. I definitely see room where group work could have been used to deal with some of the cases instead of me first bringing them out in large class discussion

Rational Expressions Quiz
Mr. Fahrenbacher
Math 80s

Name: [redacted]

A. Use any method to perform the indicated division.

1/3 1) $\dfrac{x^5+x^3+x^2+1}{x^2-4}$

1.5/2 2) $\dfrac{x^4+4x^3-23x^2+7x-3}{x-3}$

B. Simplify completely.

2.5/5 3) $\dfrac{x^3-27}{x-2} \div \dfrac{x-3}{(x+3)(x^2-4)}$

2/4 4) $\dfrac{x^2+3x+2}{x^2+x-6} \times \dfrac{2-x}{x^2-1}$

4/4 5) $\dfrac{(x+6)(x-4)}{x-6} - \dfrac{3(3x-10)}{x-6}$

3/4 6) $\dfrac{2}{x^2+4x+3} + \dfrac{-7}{x^2-x-12}$

2/3 7) $\dfrac{2+\dfrac{x}{x+3}}{3+\dfrac{x-1}{x+3}}$

C. Use partial fractions to split the fraction into two fractions.

4/4 8) $\dfrac{10x+26}{(x+5)(x-3)}$

(14)

Rational Expression Quiz Scoring Rubric

Name: [redacted]
Score: 20 /29

1) 1 point - Set up long division — 1
1 point - Inserted place holder 0's — 0
1 point - Logical Answer — 0
Total: **3 points** — 1

2) 1 point - Set up long division or synthetic division correctly — 1
1 point - Logical Answer — 0.5
Total: **2 points** — 1.5

3) 1 point - Factored difference of cubes correctly — 0
1 point - Factored difference of squares correctly — 0
1 point - Correctly inverted and multiplied — 1
1 point - Completely canceled — 1
1 point - Logical Answer — 0.5
Total: **5 points** — 2.5

4) 1 point - Factored difference of squares correctly — 1
1 point - Factored out negative correctly — 0
1 point - Completely canceled — 0.5
1 point - Logical Answer — 0.5
Total: **4 points** — 2

5) 1 point - Distributed negative correctly — 1
1 point - Factored correctly — 1
1 point - Completely canceled — 1
1 point - Logical Answer — 1
Total: **4 points** — 4

6) 1 point - Factored denominators correctly — 1
1 point - Found common denominator correctly — 1
1 point - Factored and Canceled correctly — 0.5
1 point - Logical Answer — 0.5
Total: **4 points** — 3

7) 1 point - Eliminated internal denominators correctly — 1
1 point - Factored and Canceled correctly — 0.5
1 point - Logical Answer — 0.5
Total: **3 points** — 2

8) 1 point - Set up equation correctly — 1
1 point - Eliminated denominators correctly — 1
1 point - Found "A" and "B" values correctly — 1
1 point - Logical Answer — 1
Total: **4 points** — 4