ELLIPTIC CURVES AND ICOSAHEDRAL GALOIS REPRESENTATIONS

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In 1917, Hecke proved that one-dimensional complex Galois representations give rise to entire L-series. In the 1930's, Artin conjectured that a generalization of such a result should be true: n-dimensional irreducible complex projective representations of finite Galois groups should also give rise to entire L-series.

The case of one-dimensional Galois representations was proved in full generality with the advent of Class Field Theory, and soon thereafter researchers began work on the irreducible two-dimensional representations. Klein showed that the only finite images in $PGL_2(\mathbb{C})$ correspond to regular polygons and polyhedra. Most of these cases of the conjecture have been answered in the affirmative: Irreducible cyclic and dihedral representations are induced from one-dimensional ones, while irreducible tetrahedral and octahedral representations were studied by Langlands and Tunnell. The first known example to verify Artin's conjecture for the icosahedral case did not surface until Buhler's work in 1977.

One approach to the icosahedral case starts by realizing an icosahedral Galois extension $K/\mathbb{Q}$ as one which is contained in the field generated by the 5-torsion of an elliptic curve. The overall goal of this thesis is to show how the icosahedral representation constructed by Buhler is attached to a modular elliptic curve. Using the work of Klein, we find a $\mathbb{Q}$-curve E_B defined over $\mathbb{Q}(\sqrt{5})$ such that $K(\zeta_5) = \mathbb{Q}(E_B[5]_x)$ is the field generated by the x-coordinates of the 5-torsion, where $K/\mathbb{Q}$ is the icosahedral extension studied in Buhler's work. We also find a weight 2 modular form over $\mathbb{Q}$, associated to E_B which is a deformation of Buhler's weight 1 modular form.

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