SUMMARY OF PUBLICATION CONTENT

1. Articles on Galois Representations

On the Modularity of Wildly Ramified Galois Representations
Submitted

> We show that an infinite family of odd complex 2-dimensional Galois representations ramified at 5 having nonsolvable projective image are modular, thereby verifying Artin's conjecture for a new case of examples. Such a family contains the original example studied by Buhler. In the process, we prove that an infinite family of residually modular Galois representations are modular by studying Λ-adic Hecke algebras.

Extending the Serre-Faltings Method for $\mathbb{Q}$-Curves
In preparation

> We consider a method for calculating modular forms associated to elliptic curves with a rational point of order $\ell = 2, 3$. We discuss a variant of the Serre-Faltings method which considers the symmetric square representations. As an application, we show that certain $\mathbb{Q}$-curves with reducible mod 3 representations are modular.

Icosahedral $\mathbb{Q}$-Curve Extensions
Mathematical Research Letters, 10 (2003), no. 2-3

> We consider elliptic curves defined over $\mathbb{Q}(\sqrt{5})$ which are either 2- or 3-isogenous to their Galois conjugate and which have an absolutely irreducible mod 5 representation. Using Klein's classical formulas which associate an icosahedral Galois extension $K/\mathbb{Q}$ with the 5-torsion of an elliptic curve, we prove that there is an association of such extensions generated by quintics $x^5 + A\,x^2 + B\,x + C$ satisfying $A\,B = 0$ with the aforementioned elliptic curves.

Artin's Conjecture and Elliptic Curves
Council for African-Americans in the Mathematical Sciences, Vol. III; Contemp. Math. 275 (2001)

> Artin conjectured that certain Galois representations should give rise to entire L-series. We give some history on the conjecture and motivation of why it should be true by discussing the one-dimensional case. The first known example to verify the conjecture in the icosahedral case did not surface until Buhler's work in 1977. We explain how this icosahedral representation is attached to a modular elliptic curve isogenous to its Galois conjugates, and then explain how it is associated to a cusp form of weight 5 with level prime to 5.

2. Articles on Elliptic Curves

Heron Triangles via Elliptic Curves
with Davin Maddox. Rocky Mountain Journal of Mathematics (To appear)

> Given a positive integer n, one may ask if there is a right triangle with rational sides having area n. Such integers are called congruent numbers, and are closely related to elliptic curves in the form $y^2 = x^3 - n^2 x$. In this paper, we generalize this idea and show that there is a correspondence between positive integers n associated with arbitrary triangles with rational sides having area n and the family of elliptic curves $y^2 = x\,(x - n\,\tau)\,\left(x + n\,\tau^{-1}\right)$ for nonzero rational τ.

Explicit Descent via 4-Isogeny on an Elliptic Curve
Submitted

> We work out the complete descent via 4-isogeny for a family of rational elliptic curves with a rational point of order 4; such a family is of the form $y^2 + x\,y + a\,y = x^3 + a\,x^2$ where $\sqrt{-a} \in \mathbb{Q}^\times$. In the process we (1) exhibit the 4-isogeny and the isogenous curve, (2) explicitly present the principal homogeneous spaces, and (3) discuss examples by computing the rank.

Heron Triangles, Diophantine Problems and Elliptic Curves
with Garikai Campbell. Submitted

> For all nonzero rational t, $E_t : v^2 = u^3 + (t^2 + 2)\,u^2 + u$ is an elliptic curve defined over $\mathbb{Q}$. By analyzing this family of curves, we are able to describe connections between the problem of finding Heron triangles with a given area possessing at least one side of a particular length, finding points in the plane at rational distance, and finding Diophantine quadruples and quintuples. We are then, quite naturally, led to study the relationship between these problems and elliptic curves defined over $\mathbb{Q}$ with rational torsion subgroup equal to $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/8\mathbb{Z}$. Consequently, we find a new elliptic curve with this torsion having rank 3 (tying the record for the largest known rank of an elliptic curve of this kind). Assuming the Parity Conjecture, we also find several others with rank 1 or 3 and a few more having rank 2 or 4. We also find an infinite family of elliptic curves defined over $\mathbb{Q}$ with rational torsion subgroup equal to $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/8\mathbb{Z}$ and rank at least 1 (again tying the record).

3. Miscellaneous Articles

Pythagorean Quadruplets
with Alain Togbe. Submitted

> We consider the multplicative properties of integer quadruplets (a, b, c, d) satisfying $a^2 + b^2 + c^2 = d^2$ as a generalization of Pythagorean Triplets. In the proces we present a group structure on the rational points on the unit sphere minus the poles and discuss a factorization result.

A Ternary Algebra with Applications to Binary Quadratic Forms
Council for African-Americans in the Mathematical Sciences, Vol. IV; Contemp. Math. 284 (2001)

We discuss multiplicative properties of the binary quadratic form $a\,x^2 + b\,x\,y + c\,y^2$ by considering a ring of matrices which is closed under a triple product. We prove that the ring forms a ternary algebra in the sense of Hestenes, and then derive both multiplicative formulas for a large class of binary quadratic forms and a type of multiplication for points on a conic section which generalizes the algebra of rational points on the unit circle.

On the Distribution of Fractional Parts
with Mel Currie. Council for African-Americans in the Mathematical Sciences, Vol. III; Contemp. Math. 275 (2001)

Let $[\cdot]$ denote the greatest integer function. Then

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{n}{k}-\left[\frac{n}{k}\right]\right)\frac{1}{n}=1-\gamma,$$

where $\gamma = 0.57721566\ldots$ is the Euler-Mascheroni constant. We generalize to results for the greatest multiple of $\frac{1}{m}$ and nearest multiple of $\frac{1}{m}$. In addition, we examine the limiting distribution of the fractional parts, establishing along the way connections with Stirling's approximation as well as the gamma and digamma functions. A measure μ associated with the distribution satisfies

$$\left[\mu\left[0,\mu\left[1-\frac{1}{m},1\right]\right]\right]_m=\frac{1}{m},$$

where $[x]_m$ denotes the greatest multiple of $\frac{1}{m}$ in x.

4. Doctoral Thesis

Elliptic Curves and Icosahedral Galois Representations
Stanford University (1999)

In 1917, Hecke proved that one-dimensional complex Galois representations give rise to entire L-series. In the 1930's, Artin conjectured that a generalization of such a result should be true: n-dimensional irreducible complex projective representations of finite Galois groups should also give rise to entire L-series.

The case of one-dimensional Galois representations was proved in full generality with the advent of Class Field Theory, and soon thereafter researchers began work on the irreducible two-dimensional representations. Klein showed that the only finite images in $PGL_2(\mathbb{C})$ correspond to regular polygons and polyhedra. Most of these cases of the conjecture have been answered in the affirmative: Irreducible cyclic and dihedral representations are induced from one-dimensional ones, while irreducible tetrahedral and octahedral representations were studied by Langlands and Tunnell. The first known example to verify Artin's conjecture for the icosahedral case did not surface until Buhler's work in 1977.

One approach to the icosahedral case starts by realizing an icosahedral Galois extension $K/\mathbb{Q}$ as one which is contained in the field generated by the 5-torsion of an elliptic curve. The overall goal of this thesis is to show how the icosahedral representation constructed by Buhler is attached to a modular elliptic curve. Using the work of Klein, we find a $\mathbb{Q}$-curve E_B defined over $\mathbb{Q}(\sqrt{5})$ such that $K(\zeta_5) = \mathbb{Q}(E_B[5]_x)$ is the field generated by the x-coordinates of the 5-torsion, where $K/\mathbb{Q}$ is the icosahedral extension studied in Buhler's work. We also find a weight 2 modular form over $\mathbb{Q}$, associated to E_B which is a deformation of Buhler's weight 1 modular form.