Washington University in St. Louis
SCHOOL OF ENGINEERING & APPLIED SCIENCE

ESE 553 - Nonlinear Dynamical Systems

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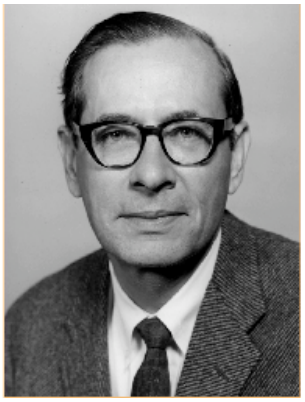
Definition (Birkhoff) Suppose $\dot{x} = f(x)$, where f is C^1 on an open set $\mathcal{O}$. If x_0 is an initial condition and

$$\bar{x} = lim_{t_n \to \infty} \phi(t_n, x_0)$$

then $\bar{x}$ is an ω-limit point of x_0. If $t_n \to -\infty$ then $\bar{x}$ is an α-limit. The set of such points is denoted by $\omega(x_0)$ and $\alpha(x_0)$.

G. D. Birkhoff

LaSalle's Invariance Theorem Consider $\dot{x} = f(x)$ where f is C^1 on an open set U and $x_0 \in U$. Suppose $V : U \to \mathbb{R}$ is C^1 and bounded from below. If $\phi(t, x_0)$ is bounded from below and $\dot{V}(\phi(t, x_0)) \leq 0$, then $\omega(x_0) \subset \dot{V}^{-1}(0)$.



J. P. LaSalle

Barbosov-Krasovski Theorem Consider $\dot{x} = f(x)$ where f is C^1 on $\mathbb{R}^n$. Suppose $V : \mathbb{R}^n \to \mathbb{R}$ is C^1 and

1. V is positive definite,
2. $\dot{V}$ is negative definite, and
3. If $\|x\|x \to \infty$ the $V(x) \to \infty$.

Then, the origin is globally asymptotically stable. that is ,) is stable and $\phi(t, x_0) \to 0$ as $t \to \infty$ for all $x_0 \in \mathbb{R}^n$.

Theorem Suppose $x_0 \in U$ and $V : U \to \mathbb{R}$ is continuous and positive on $U - \{x_0\}$, with $V(x_0) = 0$. For small enough c, $V^{-1}(c) \cap U$ is a compact neighborhood of x_0.