Washington University in St. Louis

School of Engineering & Applied Science

ESE 553 - Nonlinear Dynamical Systems

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A Converse Theorem. Suppose $\dot{x} = f(x), f(0) = 0$ where f is C^1 on an open set $0 \in \mathcal{O}$. If there exists $k, \gamma > 0$ so that for every initial condition $x_0 \in \mathcal{O}$

$\|\phi(t, x_0)\| \leq k\mathrm{e}^{-\gamma t}\|x_0\|,$

then there eixsits a ball $B_\epsilon(0) \subset \mathcal{O}$ and a smooth Lyapunov function V defined on $B_\epsilon(0)$ satisfying

1. $c_1\|x\|^2 \leq V(x) \leq c_1\|x\|^2$
2. $\dot{V} \leq -c_3\|x\|^2$
3. $\|\frac{\partial V}{\partial x}\| \leq c_4\|x\|$

Massera's Theorem Consider $\dot{x} = f(x)$, where $f : U \to \mathbb{R}^n$ is C^1 and $f(x_0) = 0$. Suppose U is positively invariant and x_0 is asymptotically stable on U. Then there exists a C^1 function $V : U \to \mathbb{R}$ for which V is positive definite and $\dot{V}$ is negative definite on U.

Persistence of equilibria

Theorem Suppose $\dot{x} = f(x, \mu)$ is C^1 in $(x, \mu) \in U \times V \subset \mathbb{R}^n \times \mathbb{R}^n$ and $f(x_0, \mu_0) = 0$.

If $\sigma(\frac{\partial f}{\partial x}|_{(x_0,\mu_0)}) \subset \mathbb{C}^-$, then there exists $\epsilon > 0$ such that for $\|\mu - \mu_0\| < \epsilon$ there is a smooth branch of asymptotically stable equilibria $x(\mu)$ for which $x(\mu_0) = x_0$.

A population model with limits to growth and constant harvesting

$$\dot{x} = x(1 - x) - h$$

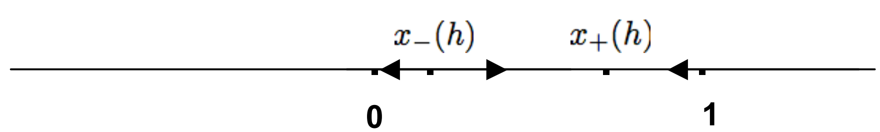
Equilibrium equations:

$$x^2 - x + h = 0$$

$$x_{\pm}(h) = \frac{1 \pm \sqrt{1 - 4h}}{2}$$

Equilibria exists for $0 \leq h \leq 1/4$ and satisfy: $0 \leq x_{\pm}(h) \leq 1.$

For $0 \leq h \leq 1/4$ and $|x| >> 0, \quad \dot{x} < 0$



N.B. **Extinction** occurs if h > 1/4. At h = 1/4, the dynamical system undergoes a **bifurcation**. To say h > 1/4 is to say, in our original model, that u > 4aN.