Washington University in St. Louis
School of Engineering & Applied Science

ESE 441 - Control Systems

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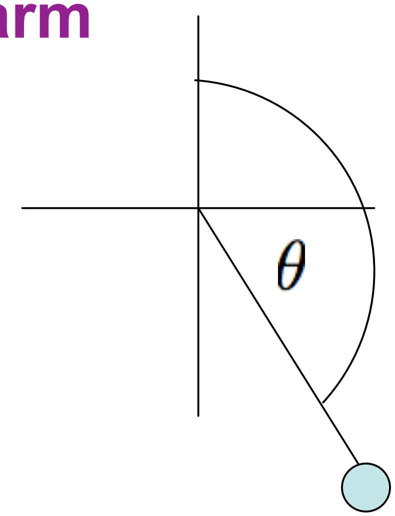
A revison to Lecture 9, slides 3-9

Control of a robot arm

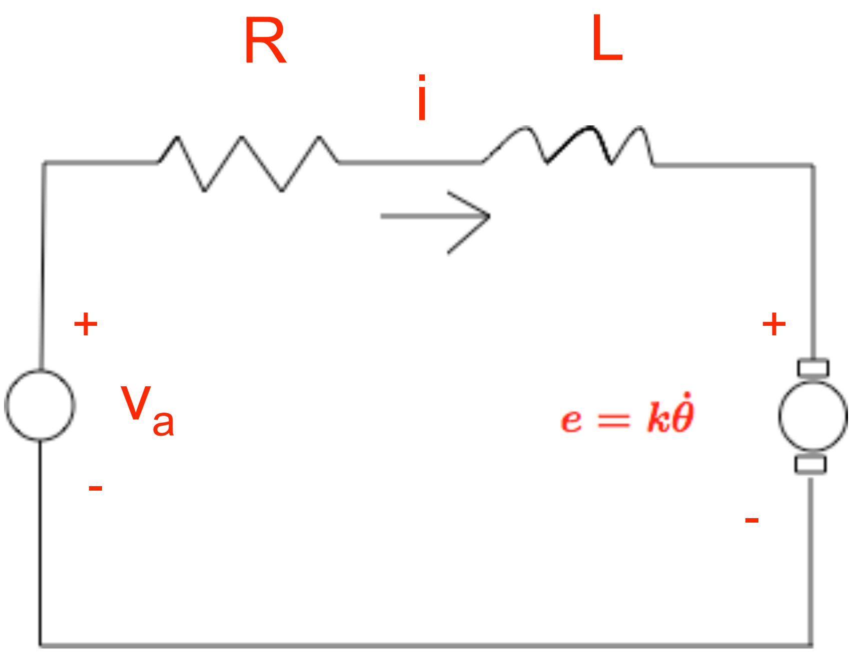


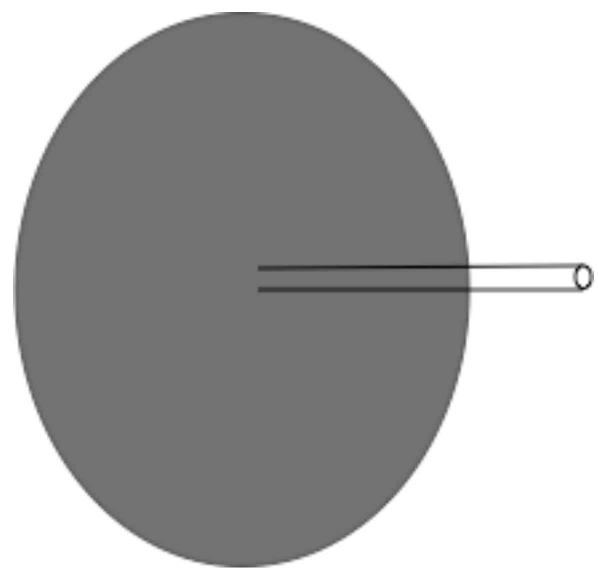
Torque balance equation

$$J\ddot{\theta} + F\dot{\theta} = T = Ki = u$$

Kirchhoff's voltage law:

$$v_a = Ri + L\frac{di}{dt} + k\frac{d\theta}{dt}$$





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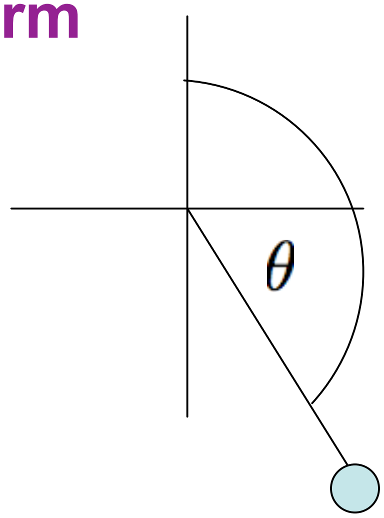


Torque balance equation:

$$J\ddot{\theta} + F\dot{\theta} = T = Ki = u$$

Kirchhoff's voltage law:

$$v_a = Ri + L\frac{di}{dt} + k\frac{d\theta}{dt}$$

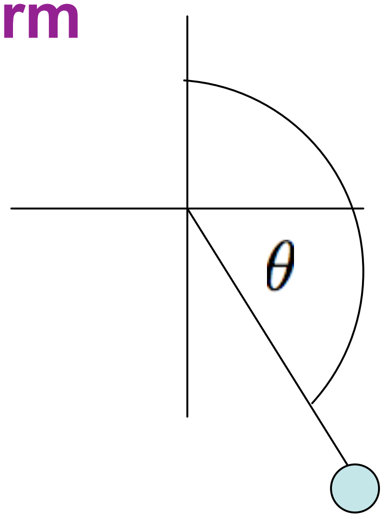
Set $v_a = 0$.

Is this uncontrolled system asymptotically stable?

These coupled equations can be expressed as a first order vector equation:

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} i \\ \theta \\ \dot{\theta} \end{pmatrix} \qquad \dot{x} = Ax$$

Control of a robot arm



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$$v_a = Ri + L\frac{di}{dt} + k\frac{d\theta}{dt}$$

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$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} i \\ \theta \\ \dot{\theta} \end{pmatrix}$$

$$\dot{x} = Ax$$

where

$$\dot{x} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} -R/L & 0 & -k/L \\ 0 & 0 & 1 \\ K/J & 0 & -F/J \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = Ax$$

Control of a robot arm

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The eigenvalue equation is

$$Ax = \lambda x$$

or

$$\begin{pmatrix} -R/L & 0 & -k/L \\ 0 & 0 & 1 \\ K/J & 0 & -F/J \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \lambda x_1 \\ \lambda x_2 \\ \lambda x_3 \end{pmatrix}$$

$$R = 1, L = 2, J = 1, F = 2, k = 1, K = 1$$

Control of a robot arm

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For example, if

$$R = 1, L = 2, J = 1, F = 2, k = 1, K = 1$$

The eigenvalue equation is

$$\begin{pmatrix} -.5 & 0 & -.5 \\ 0 & 0 & 1 \\ 1 & 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

This system is not asymptotically stable, because one of its eigenvalues is 0.

We can find the eigenvalues and eigenvectors using MATLAB.

```
>> A= [-.5 0 .5; 0 0 1; 1 0 -2]

A =

   -0.5000         0    0.5000
         0         0    1.0000
    1.0000         0   -2.0000

>> eig(A)

ans =

         0
   -0.2192
   -2.2808
```

We can find the eigenvalues and eigenvectors using MATLAB.

```
>> [X,D] = eig(A)

X =

         0    0.3563   -0.2490
    1.0000   -0.9127   -0.3889
         0    0.2001    0.8870


D =

         0         0         0
         0   -0.2192         0
         0         0   -2.2808
```