

Hints for Problem 6.4, p. 198 in the Text.

First of all, the problem should have also included the output  $y = Cx$ , where  $C \neq 0$ .

Next, in systems theoretic terms, the problem is to show that the DC gain of the closed-loop transfer function from the disturbance  $d$  to the output  $y$  is 0.

Hints: Start by writing down the system

$$(0.1) \quad \dot{x} = Ax + Bu + Fd$$

$$(0.2) \quad \dot{z} = Cx$$

Since the pair  $\begin{pmatrix} A & 0 \\ C & 0 \end{pmatrix}, \begin{pmatrix} B \\ 0 \end{pmatrix}$  is controllable, there exists a state feedback law

$$u = -K_1x - K_2z$$

such that the closed-loop system

$$(0.3) \quad \dot{x} = Ax - BK_1x - BK_2z + Fd$$

$$(0.4) \quad \dot{z} = Cx$$

with input  $d$  and output  $y$  is asymptotically stable.

1. Compute the closed-loop transfer function  $G(s)$ .
2.  $G$  has its poles in the LHP, and
3.  $G(s)$  has a factor of  $s$  in its numerator.

To understand the formula for  $G$ , I found it useful to write out the eigenvalue equation

$$\begin{pmatrix} A - BK_1 & -BK_2 \\ C & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \lambda \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

in terms of  $v_1$ , by eliminating  $v_2$ .