Exam #1 - Take-home exam due Tuesday, October 28, 2008 by 11:30 AM. The exams will be collected in the ESE441 bin in Bryan Hall at 11:30AM or may be handed in at the ***beginning*** of class. No exceptions will be permitted.

Ground rules: Open book, open notes, open computer and open mind, but *no* collaboration.

CIB will be available by e-mail for most of Friday morning. I will check my e-mail throughout the weekend and will be available on Monday in AB-464, for most of the day. However, do check first with me by e-mail or phone.

Good luck!

Problem 1. (25 pts) Consider the control system

$$\dot{x} = Ax + bu, \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}$$

where

$$A = \begin{pmatrix} -d_1 & -d_2 & \dots & -d_n \\ 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & 0 \\ 0 & 0 & \dots & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Suppose $n = 4$ and let $d_1, \dots, d_4$ be the first four digits of your ID number plus or minus 1, in any order you choose.

1a. Show that the resulting system is controllable, by showing that c the controllability matrix $\mathcal{C}$ is nonsingular. Use MATLAB to compute the determinant of $\mathcal{C}$ and to complete or check your calculation of $\mathcal{C}$.

1b. Write down the eigenvalue equation for A. What are the eigenvalues and eigenvectors of A? (You may use MATLAB)? What is the characteristic polynomial of A?

Using MATLAB

1c. Design a control law $u = -Kx + v$ so that the closed-loop system has $\{-1, -2. -3, -4\}$ as its eigenvalues.

1d Choose any nonzero constant γ and define the system output to be

$$y = cx$$

where

$$c = (\gamma, 0, \ldots, 0)$$

What is the DCgain of your system?

1e. Using MATLAB, compute and graph the impulse response function of your system. Is your system asymptotically stable? Why? Compute and graph the step response of your system. Does your answer agree with your answer to 1c.? Why?

2. (25 pts) Choose three positive numbers and call them λ, ρ, σ. Now consider the matrix

$$A = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \rho & \sigma \\ 0 & -\sigma & \rho \end{pmatrix}$$

Without using MATLAB

2a. Compute the characteristic polynomial of A.

2b. What are the eigenvalues of A?

2c. Is the system $\dot{x} = Ax$ asymptotically stable?

2d. Compute e^{At}.

2e. What are the eigenvalues of e^{At}?

Problem 3. Consider the "triple integrator"

$$\frac{d^3x}{dt^3} = u$$

3a.(2 pts) Show that the system is controllable to 0.

3b.(8 pts) Find a time T and a control law $u(t), \quad 0 \leq t \leq T$ that drives the initial state

$$x_0 = \begin{pmatrix} \lambda \\ \rho \\ \sigma \end{pmatrix}$$

to 0 in time T, where λ, ρ, σ are the constants you used in Problem 2.

Problem 4.(10pts) Consider the control system

$$\dot{x} = Ax + bu,\ y = cx,\ \ x \in \mathbb{R}^3,\ u, y \in \mathbb{R}$$

where A is the matrix you used in Problem 2, and where

$$b = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix},\ \ c = (1\ 0\ 2)$$

Design a control law $u(t)$ so that the steady-state response $y_{ss}(t)$ satisfies

$$y_{ss}(t) = \sin(t)$$

Problem 5.(10pts) Problem 5.7 in the text.

Problem 6.(10pts) Problem 6.8 in the text.

Problem 4.(10pts) Problem 6.9 in the text.