**HW5. Due November 11, 2008 at 11:30 AM.**

**Problem 1. (25 pts) Consider the control system**

$$\dot{x} = Ax + bu, \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}$$

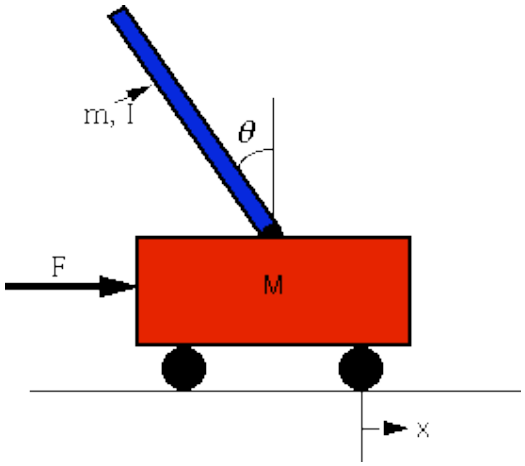
**where**

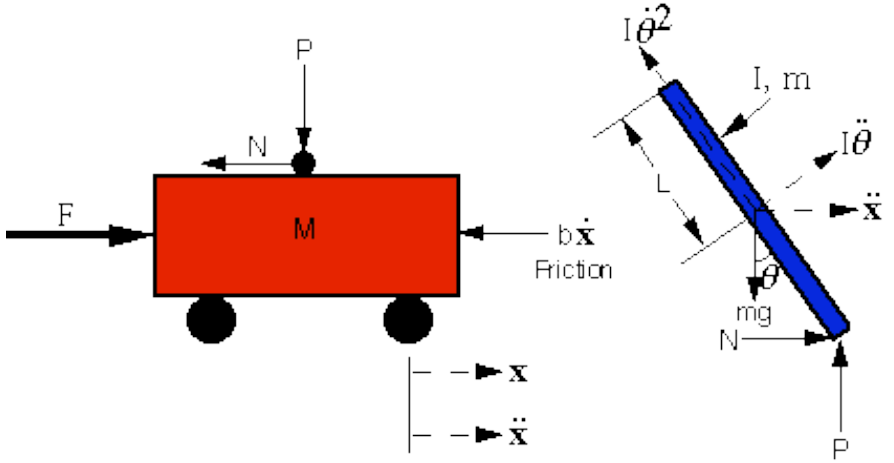
$$A = \begin{pmatrix} -d_1 & -d_2 & \dots & -d_{n-1} & -d_n \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

**Show that this system is controllable.**

**Problem 2. (25 pts) Problem 7.1, on page 226 of the text.**

# Problem 3 (25 pts). Consider the Cart and Pendulum




Torque Balance Equation

$$m\ddot{x} + ml\ddot{\theta} - mg\theta = 0$$

Horizontal Force Balance Equation

$$(M+m)\ddot{x} + ml\ddot{\theta} - F = 0$$

Set M = 1, m = .I , *l* = 1, g = 10

## Problem 3a. (5 pts)

If z(t) = horizontal displacement of the cart, complete the state space equations

$$\dot{x} = Ax + bu$$
$$y = cx$$

where

$$x = \begin{pmatrix} z \\ \dot{z} \\ \theta \\ \dot{\theta} \end{pmatrix} \quad \text{and} \quad A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 11 & 0 \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

by specifying y = $\theta$ as the output.

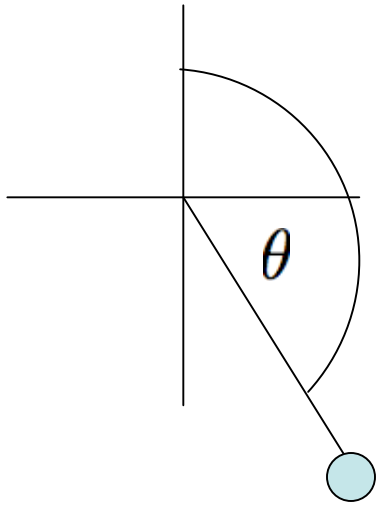
**Problem 3b. (10 pts)** For the system with this output, compute the observability matrix.

**Problem 3c. (5 pts)** Is the system observable? If not, find an initial state that is unobservable.

**3d. ( 5 pts.)** Explain your answer to problem 2c in terms of the physical states of the cart and the pendulum.

Problem 4 (25pts) Consider the robot arm:

$$J\ddot{\theta} + F\dot{\theta} = T = Ki = u$$




$$x = \begin{pmatrix} \theta \\ \dot{\theta} \end{pmatrix} \qquad \dot{x} = Ax + bu \qquad y = cx$$

$$A = \begin{pmatrix} 0 & 1 \\ 0 & -F/J \end{pmatrix} \qquad b = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad c = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

Design a state feedback control law $u = -Kx + kr$ so that the closed-loop system

1. is asymptotically stable
2. is lightly underdamped
3. has DC gain = 4
4. Has a 2% settling time $T_s = 10$