

HW # 3: Due Tuesday, October 9, 2008 by class time.

1. (25 pts.) Compute e^{At} for each of the following matrices A:

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} \quad \begin{pmatrix} -1 & 1 \\ 0 & -2 \end{pmatrix} \quad \begin{pmatrix} -1 & 0 \\ 1 & -2 \end{pmatrix}$$

2. (25 pts.) Determine the solution of the differential equation

$$\dot{x} = \begin{pmatrix} -1 & 2 \\ -2 & -1 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Compute the characteristic polynomial of this matrix.

What are its roots?

Is this system asymptotically stable? Why or why not?

3. (25 pts.) Consider the ODE

$$\ddot{y} + 2\dot{y} + 5y = 0$$

What is its characteristic equation?

What are its roots?

Is the system asymptotically stable? Why or why not?

4. (25 pts.) Consider the system

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

What is the solution for the initial condition

$$x_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}?$$

Is the system asymptotically stable? Why or why not?

EXTRA CREDIT PROBLEM (25 pts.)

Give the expression for e^A for

a. $A = \begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix}$

b. $B = \begin{pmatrix} 0 & b \\ -b & 0 \end{pmatrix}$

c. $A + B$.

d. Compute e^{A+B}

e. Show $e^{A+B} = e^A e^B$ (Hint: show $AB = BA$)

What are the (1, 1) and the (1, 2) entries of this identity?