Washington University in St. Louis
SCHOOL OF ENGINEERING & APPLIED SCIENCE

ESE 441 - Control Systems

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Linear change of coordinates

- Eigenvalues are invariant
- Asymptotic stability is invariant
- DC Gain is invariant
- Frequency response is invariant

Transformation to simpler form for system equations

- Eigenvalue Assignment
  - Controllability is invariant
  - Ackermann’s Formula

Eigenvalue assignment and Ackermann's Formula for reachable systems

$$\dot{x} = Ax + bu$$

$$\det(sI - A) = s^n + d_1 s^{n-1} + \cdots + d_n$$

Does there exists a change of coordinates $z = Tx$ such that the control system takes the form

$$\dot{z} = \widetilde{A}z + \widetilde{b}u$$

where

$$\widetilde{A} = \begin{pmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -d_n & -d_{n-1} & -d_{n-2} & \ldots & -d_1 \end{pmatrix}, \quad \widetilde{b} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$

If so,

$$\widetilde{\mathcal{C}} = (\tilde{b}\ \tilde{A}\tilde{b}\ \ldots \tilde{A}^{n-1}\tilde{b}) = T(b\ Ab\ \ldots A^{n-1}b) = T\mathcal{C}$$

and

$$T = \widetilde{\mathcal{C}}\mathcal{C}^{-1}$$

Suppose $a(s) = s^n + a_1 s^{n-1} + \cdots + a_n$

is a desired closed-loop characteristic polynomial for $u = -Kx + v$

where $K = (k_n\ k_{n-1}\ \ldots k_1)$ is to be found.

For $\tilde{A}, \tilde{b}$ this is easy:

$$\widetilde{K} = (\widetilde{k_n}\ \widetilde{k_{n-1}}\ \ldots \widetilde{k_1}) = (a_n - d_n\ \ a_{n-1} - d_{n-1}\ \ldots\ \ a_1 - d_1)$$

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and therefore

$$Kx = \widetilde{K} T x = \widetilde{K} \widetilde{\mathcal{C}} \mathcal{C}^{-1} x$$

leading to Ackermann's formula

$$K = \widetilde{K} \widetilde{\mathcal{C}} \mathcal{C}^{-1}$$

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- e^{At}
  - Jordan normal form