ESE 441 - CONTROL SYSTEMS

Exam 2: DUE IN THE ESE 441 BIN BY 4:30 PM, DECEMBER 11, 2008

Ground rules: Open book, open notes, open computer and open mind, but *no* collaboration.

If you use MATLAB for a part of any problem, please submit your code as well as your numerical answers and your figures.

Take the first two digits of your SSN, add them and set c = last digit or 1 if this is 0. Take the last three digits of your SSN, add them and set d = last digit or 2 if this is 0.

1. Consider the two dimensional control system

$$\dot{x} = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} x + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u \qquad y = (1\ \ 0)x$$

1a. (5 pts) Compute the eigenvalues of A. Show that b is an eigenvector of A.

1b. (5pts) Is the system controllable?

1c. (5pts) Is the system observable?

1d. (5pts) Using the “place” command in MATLAB find a state feedback law placing the closed-loop eigenvalues at (-c-2,-3). At (- 3,-d - 4).

1e. (5pts) Compute the system transfer function. Show you can place the eigenvalues in 1d using constant gain output feedback, justifying your answer using the root locus plot.

Problems 2 and 3

In class we designed a PI controller for a robot arm to hold a ball of unknown mass at a desired angular position. We want to enhance our model by including actuator dynamics, where the arm is controlled by a DC motor. This model was developed beginning in Lecture 9 where θ_m , the position of the robot arm (rotor), is the output and where v_a , the applied voltage to the DC motor, is the input. We will fix the constants as follows:

R=1, L=2, J=1, F=2, k=c, K=d.

Problem 2a. (5pts) Find the the transfer function g(s) from v_a to θ_m

2b. (5pts) Using MATLAB give the root-locus plot for g.

2c. (5pts) For what values of the constant output feedback gain k (for $k>0$) is the closed-loop systems stable?

Problem 3a. (5pts) Find a state-space realization for the transfer function g.

3b. (10pts) Design a state space feedback/estimation scheme placing the controller eigenvalues at (-c-1, -d-1, -3) and the observer eigenvalues at (-2c-2, -2d -2, -6).

3c. (5pts) Express the resulting six-dimensional closed-loop System in (x,e) coordinates and justify the stability of the overall closed-loop system.

3d. (5pts) Using MATLAB, plot the impulse response function of the overall closed-loop system.

#4. (10 pts.) For which of the following six transfer functions will there exist an integral controller k_I/s, with $k_I > 0$, so that the closed-loop system will be stable and have DC gain 0?

$$g(s) = (s \pm 2)/(s^2 + .5s + 1)$$

$$g(s) = (s \pm 2)/(s^2 + 2s + 1)$$

$$g(s) = (s \pm 2)/(s^2 + 10s + 1)$$

5. For one of the choices for which you answered #3 in the affirmative, design a gain k_I for which the closed-loop system is stable. (6pts)

Show that the DC gain is 0. (2pts)

Using MatLab, illustrate your answer by graphing the closed-loop step response. (2 pts.)

6. In class, using constant gain output feedback we stabilized the rational function

$$g_{OL}(s) = (-30s + 180)/s(s^2 + 4s + 13),$$

which models the transfer function form the elevator to the altitude of a Boeing 747*. For this feedback law, the rise time for a unit step repsonse was ~ 20 seconds.

6a. (3 pts) Find a 3-dimenional state-space realization of g_{OL}

6b. (2 pts) Check that the realization is both controllable and observable

6c. (10 pts.) Design a state space feedback/estimation scheme stabilizing the closed-loop system and having a unit step response with rise time less than 3 seconds.

*FYI, this model is developed on pages 124-125 and more fully on pages 742-760 of **Feedback Control of Dynamic Systems**, by Franklin, Powell and Emami-Naeni, Prentice-Hall, 5th Edition, 2006. *You don't need anything but the transfer function to do Problem 6.*