

Activity #1: Introduction to Probability & Statistical Inference

Suppose your statistics professor on the first day of class proclaims:

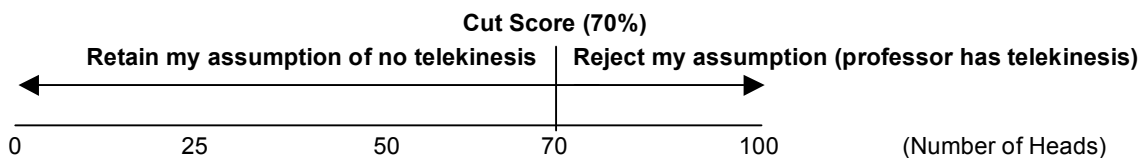
“I have developed the power of mind over matter. I will stand up front while one student tosses 100 pennies in the back of the room. Through the sheer power of my mind, I will cause those pennies to come up heads. Now I admit that I have not yet perfected this skill – I cannot make **all** the tosses come up heads. What I can do is produce an **impressive** number of heads, to the point where you will believe that I may be telling the truth.”

1) Suppose this actually happens in our class. How many heads would have to turn up in the 100 tosses before you would find the results “impressive” enough to take my claim seriously?

2) Would you be impressed if the student tossed 50 heads? Why or why not?

I expect that you would not be impressed with 51, 52, or 53 heads out of 100 tosses, but what about 65, 75, or 85 heads? At what point would a rational, open-minded person reasonably say, “I am impressed?” Suppose that after reflecting on this question, you decide that you would be impressed if 70% or more of the pennies turn up heads. Inside your mind, you decide:

1. I will assume that nothing special is happening. In this experiment, I will assume that the professor has no special ability
2. I will watch the penny tosses – continuing under the belief that my assumption is true
3. Under my assumption, it is highly unlikely that more than 70% of the pennies will turn up heads. If I see 70% or more of the pennies are heads, I will conclude that my assumption is false. I will conclude the professor does have special ability.



3) This type of logic is called statistical inference (or sometimes *Statistics*). What is the difference between statistics and Statistics? What is probability?

statistics:

Statistics:

Probability:

4) Suppose we conduct our experiment and the student tosses 52 heads. What do we conclude? Are we sure of our conclusion?

5) Assuming the coin is fair and that the professor does not have telekinesis, what is the probability that the student will toss 50, 52, 70, or 87 heads? We'll learn how to calculate these probabilities in the first few weeks of class – for now, see how accurate your intuition is.

$P(\text{exactly 50 heads}) = \underline{\hspace{2cm}}$ $P(50 \text{ or more heads}) = \underline{\hspace{2cm}}$ $P(50 \text{ or more heads}) = \underline{\hspace{2cm}}$

$P(52 \text{ or more heads}) = \underline{\hspace{2cm}}$ $P(70 \text{ or more heads}) = \underline{\hspace{2cm}}$ $P(87 \text{ or more heads}) = \underline{\hspace{2cm}}$

6) Let's say we conduct this experiment and the student does toss more than 70 heads. According to our decision rule, we will now conclude that the professor does indeed have telekinetic abilities. How certain are we in our conclusion? What is the probability that we make an error in our conclusion?

7) This time, suppose the student tosses the penny only 10 times. Would you still draw the line at 70%? Would you be as impressed with 7 heads out of 10 tosses as you were with 70 heads out of 100 tosses?

No – this seems easier to accomplish. If we again assume that the professor does not have telekinesis, then

$P(\text{exactly 5 heads out of 10 tosses}) = 0.246$

$P(6 \text{ or more heads out of 10}) = 0.377$

$P(7 \text{ or more heads out of 10 tosses}) = 0.172$

$P(9 \text{ or more heads out of 10 tosses}) = 0.0107$

8) Let's say the student tosses 10 pennies and rolls 7 heads. Should we reject our assumption and conclude the professor does have telekinesis? Explain your reasoning.

Our goal in this class will be to learn how to analyze data in order to make effective decisions. To do this, we will need to understand probability, descriptive statistics, and inferential statistics (MATH 300). Once we understand these concepts, we can use a variety of methods to analyze all sorts of data (MATH 301).

Scenario: Rosen, B. & Jerdee, T. (1974). Influence of sex role stereotypes on personnel decisions. *Journal of Applied Psychology*, 59: 9-14.

In 1972, 24 male bank supervisors were each given the same personnel file and asked to judge whether the person (applicant) should be promoted to a branch manager position. All the supervisors were given the same personnel file, but supervisors were randomly assigned into one of two groups:

Group A: The 12 supervisors in this group were told that the applicant was **male**

Group B: The 12 supervisors in this group were told that the applicant was **female**

Of the 24 personnel files reviewed, 18 were selected for promotion.

We'll get to the results of the study in a bit. First, let's answer some questions.

- 9) When we randomly assign subjects to groups, we say the study is *experimental* (instead of *observational*). Why did the researchers randomly assign supervisors to the two groups? Why didn't they just put the oldest 12 supervisors in the first group?

The results of this experiment can be summarized in a contingency table:

Observed Data	Promoted	Not Promoted	Total
Group A (Male)			
Group B (Female)			
Total			24

- 10) What proportion of "male" and "female" applicants were promoted? What proportion of all applicants were promoted? From these results, can we conclude that male bank supervisors are sexist?

- 11) Let's assume that gender had **no impact** on promotion decisions. We'll call this our *null hypothesis* (a statement that, in essence, says "nothing happens"). If we assume this null hypothesis is true, how many promotions should we expect for our male and female applicants?

Observed Data	Promoted	Not Promoted	Total
Group A (Male)	11	1	12
Group B (Female)	7	5	12
Total	18	6	24

"Expected" Data	Promoted	Not Promoted	Total
Group A (Male)			12
Group B (Female)			12
Total	18	6	24

So our **null hypothesis** is: Gender (groups) had no impact on promotion decisions.
Under this null hypothesis, we'd expect 9 promotions (75%) in each group.

Now we can ask the key question in *statistical inference*:

Assuming our null hypothesis is true, how likely were we to get the results we actually obtained in our study?

We observed 11 male promotion decisions and only 7 female promotion decisions.

If gender had no impact on promotion decisions, how likely were we to obtain these results?

We'll learn methods to calculate probabilities in the first half of this course (*hypergeometric*) and at the end of MATH 301 (*chi-square test*). For now, we can conduct a **simulation** to estimate the likelihood in question.

SIMULATION METHODS

We will assume that supervisors make their promotion decisions randomly (gender had no impact on promotion decisions).

We'll also assume that if we could go back in time and conduct the experiment again (with supervisors likely being randomly assigned to different groups), we would still have had 18 promotion decisions and 6 non-promotion decisions.

We're interested in estimating the likelihood of observing 11 or more male promotions under these assumptions. This will tell us the likelihood of observing results as (or even more) extreme as what we actually observed.

- Take 24 index cards and write **PROMOTED** on **18** of them. Write **NOT PROMOTED** on the other **6** cards.
- Shuffle the cards and randomly select **12 cards** to represent the 12 "male" personnel files assigned to Group A.
- Count the number of promotions you get in this sample of 12 cards. Write this number in the table below.
- Repeat this simulation 5 times and complete the table

Simulation Number:	1	2	3	4	5
Number of promotions in Group A:	_____	_____	_____	_____	_____
Did you find 11 or more promotions?	_____	_____	_____	_____	_____

Combine the results from everyone in the class and complete the following table:

# of promoted males	1	2	3	4	5	6	7	8	9	10	11	12
# of repetitions												
Percentage												
% of repetitions with 11 or more male promotions												

Null hypothesis: Gender had no impact on promotion decisions

Assuming this hypothesis is true, we would expect to find 9 promoted males and 9 promoted females.

We actually observed 11 promoted males and 7 promoted females.

The likelihood of this happening, assuming the null hypothesis is true (from our simulation), is _____

Therefore, we conclude that our null hypothesis is probably: TRUE or FALSE

If you care, the actual likelihoods of obtaining a specific number of male promotions are:

Number of promoted males	Likelihood	Cumulative Likelihood
1	0.0%	0.0%
2	0.0%	0.0%
3	0.0%	0.0%
4	0.0%	0.0%
5	0.0%	0.0%
6	0.6%	0.6%
7	7.1%	7.7%
8	24.3%	32.0%
9	36.0%	68.0%
10	24.3%	92.3%
11	7.1%	99.4%
12	0.6%	100.0%

QUESTIONS:

1. Why do we assume the null hypothesis is true?
2. Suppose 10 males were promoted in this study. What's the likelihood of observing 10 or more promoted males if gender has no impact on promotion decisions?