Aiming a TV Satellite Dish

Version 1.0
23 February 2003
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Around 1945 science fiction author Arthur C. Clarke proposed that a system of three satellites in synchronous orbit with the earth could provide near-global coverage for radio communication. Each satellite would be placed in an orbit with an orbital period of exactly 24 hours, spaced equally around the earth's equator to provide near-total coverage. With such an orbital period, the satellite would appear to be stationary to an observer on the earth, maintaining a constant altitude and azimuth from any particular location on earth.

Within 20 years such "geosynchronous" satellites were in service, and today transmission power has reduced the size of the required receiver to be less than the size of a car tire in circumference. The primary task for a technician is to install the dish so that it points at the proper altitude and azimuth. This document shows the basis for the Clarke orbit and explores the geometry for pointing a receiver at a satellite directly above the observer's longitude. The derivation is suitable for a high school physics, astronomy, or advanced math class.

A. Orbital radius for a geosynchronous satellite.

To determine the distance from the center of the earth to the satellite, the first step is to recognize that the force of gravity of the earth on the satellite is the centripetal force which makes it go in a circle. Therefore,

(1) $F_c = F_g$

Substituting the definitions of centripetal force and Newton's Law of gravity for these terms, we obtain

(2) $$\frac{m_1 v^2}{r} = \frac{G m_1 m_2}{r^2}$$

where m_1 is the mass of the orbiting satellite, m_2 is the mass of the earth, v is the tangential orbital velocity of the satellite, G is the universal gravitational constant, and r is the distance from the satellite to the center of the earth (and hence the center of the circular orbit).

Cancelling terms on both sides of the equation, we obtain the following:

(3) $$\frac{v^2}{1} = \frac{G m_2}{r}$$

Which can be further simplified by noting that for a circular orbit, v = C/T, where C is the circumference of the circular orbit and T is the orbital period, the time it takes to orbit the earth once. Also, since the circumference of a circle is equal to 2πr, we can obtain this equation by substituting these definitions:

$$\left(\frac{C}{T}\right)^2 = \frac{Gm_e}{r}$$

(4) $$\left(\frac{2\pi r}{T}\right)^2 = \frac{Gm_e}{r}$$

$$\left(\frac{4\pi^2 r^2}{T}\right) = \frac{Gm_e}{r}$$

Again, combining like terms and reorganizing, we obtain this equation, known as *Kepler's Third Law of Planetary Motion.*

(5) $$r^3 = \frac{Gm_e}{4\pi^2}T^2$$

Kepler's form of this equation treated the constant $\frac{Gm_e}{4\pi^2}$ as equal to 1, using units based on the size of the earth's orbit. In the form shown in equation 5 it uses metric units and is universal for any circular orbit. (For elliptical orbits, it is the same equation, except the semi-major axis of the ellipse is used instead of the radius of the circular orbit.)

Now we can calculate the distance of a geosynchronous satellite to the center of the earth. using the following values:

T = 24 hours = 86,400 seconds

m_e= $5.94\text{x}10^{24}$ kilograms

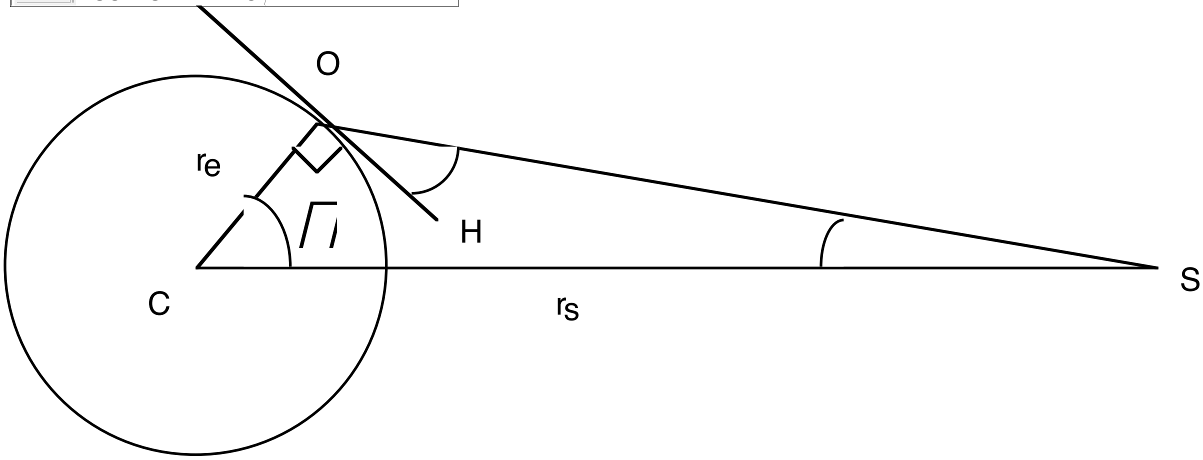
G = $6.67\text{x}10^{-11}$ Nm^2/kg^2

r = 42840150.61 m

or about 42,800 km.

Next, we need to determine the altitude angle (angle from the horizon) the satellite appears. This depends on the radius of the earth, the distance from the center of the earth t latitude as shown in this diagram.

	A	B
1	83.78444745	



In this diagram:

angle OCS is the observer's latitude, □
angle HOS is the altitude of the geostationary satellite
angle HOC is 90 degrees (by the definition of horizon)
segment OC is the radius of the earth (called r_e below)
segment CS is the distance from the center of the earth to the satellite (equal to r_s calculated earlier)

Probably the most efficient method to solve the problem is through two successive applications of the law of Cosines. For any triangle (right triangle or not),

$$b^2 = a^2 + c^2 - 2ac\cos(B)$$

where a,b,c are sides and B is the angle opposite side b.

Applying this to obtain segment OS, the distance from the observer to the satellite, we see that

$$(\overline{OS})^2 = (\overline{OC})^2 + (\overline{CS})^2 - 2(\overline{OC})(\overline{CS})\cos(\angle OCS)$$

$$(\overline{OS})^2 = r_e^2 + r_s^2 - 2r_e r_s \cos(□)$$

substituting our previously found value for r_s, plus the given value of r_e= 6378 km, and assuming a latitude of 38 degrees north (Antioch, California), we obtain this value for line segment OS:

37977610.07

meters.

Now we can apply the law of cosines again to obtain angle SOC.

$$r_s^2 = (\overline{OS})^2 + r_e^2 - 2r_e(\overline{OS})\ \cos(\angle SOC)$$

$$\cos(\angle SOC) = \left(\frac{r_s^2 - (\overline{OS})^2 - r_e^2}{2r_e(\overline{OS})}\right)$$

substituting all of our known values and solving for angle SOC, we obtain: 43.3 degrees. Note, since the diagram indicates that this angle is obtuse, the answer must be 180-43.3 degrees = 136.7 degrees; however, the altitude angle +90 degrees is angle SOC, the result must be the 136.7-90 degrees or 46.7 degrees.

Verification:

It is also now possible to verify this result using the law of sines or by using the law of cosines again to determine angle CSO. There may be more efficient ways of deriving the answer. These solutions are left as an exercise for the reader.

Conclusion:
From Antioch, CA a satellite receixver dish must have its principal angle tilted at a 46.7 degree angle from the horizontal to receive signals from a satellite fixed at 121 degrees West Longitude. Since all geosynchronous satellites are coplanar, aiming at any other satellite would simply be a matter of scanning east and west from this point using a rotation axis mounted parallel to the earth's rotation axis, as in an equatorial telescope. Modern dishes may not actually appear to be pointed in this direction as many now utilize off-axis focusing to maximize the reflecting surface of the dish. This eliminates the "shadow" cast by the antenna in older designs. In this case, the principal axis is likely to be centered on the parabolic surface of the dish, with the antenna located a fixed angle from the prinicpal axis.