AP Statistics Name__________________________

Developing Important Formulas

The purpose of the handout is to derive formulas for the mean, variance, and standard deviation of three important random variables. You should make use of rules for manipulating random variables in answering many of the questions below. The answers are given on the back of this page.

1. Let S be a random variable that only takes the values 0 or 1. It takes the value 1 with probability p and the value 0 with probability $1-p$. If the variable takes the value 1, call this a "success". If the variable takes the value 0, call this a "failure". Below is the probability distribution.

S	0	1
Probability	$1-p$	p

   (a) Find the mean (expected value) of S.
   (b) Find the variance of S. (Simplify wisely to get your answer in a nice-looking form.)
   (c) Find the standard deviation of S.

2. Reconsider the random variable S from question #1. Suppose we define a new random variable X, which is the sum of n independent values of the random variable S. Notationally, we have

$$X = S_1 + S_2 + S_3 + ... + S_n$$

   where each S_i has the distribution as defined in question #1. Think of X as representing the number of successes in n independent trials (X is thus a *binomial* random variable).

   (a) Find the mean (expected value) of X.
   (b) Find the variance of X.
   (c) Find the standard deviation of X.

3. Finally, define the random variable $\hat{p}$ as follows

$$\hat{p} = \frac{S_1 + S_2 + S_3 + ... + S_n}{n} = \frac{X}{n}$$

   Think of $\hat{p}$ as the proportion of successes in n independent trials.

   (a) Find the mean (expected value) of $\hat{p}$.
   (b) Find the variance of $\hat{p}$.
   (c) Find the standard deviation of $\hat{p}$.

ANSWERS

1. Binomial random variable with $n = 1$ (also known as a *Bernoulli* random variable).

   (a) $\mu_S = p$

   (b) $\sigma_S^2 = p(1-p)$

   (c) $\sigma_S = \sqrt{p(1-p)}$

2. Binomial random variable with n trials…sum of n independent values of S.

   (a) $\mu_X = np$

   (b) $\sigma_X^2 = np(1-p)$

   (c) $\sigma_X = \sqrt{np(1-p)}$

3. Sample proportion from n trials,

   (a) $\mu_{\hat{p}} = p$

   (b) $\sigma_{\hat{p}}^2 = \dfrac{p(1-p)}{n}$

   (c) $\sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}}$